PB81-111585

A Statistical Study of Coal Sulfur Variability and Related Factors

Foster Associates, Inc.
Washington, DC

Prepared for

Environmental Protection Agency
Research Triangle Park, NC

May 80

U.S. Department of Commerce
National Technical Information Service
NTIS

TECHNICAL REPORT DATA

(Please read Instructions on the reverse before completing)

1. REPORT NO. EPA-450/5-80-008a	2.	3. RECIPIENT'S ACCESSION NO. PB81 111585
4. TITLE AND SUBTITLE A Statistical Study of Coal Sulfur Variability and Related Factors		5. REPORT DATE May 1980 6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) George R. Warholic, John E. Morton, Yimin Ngan James E. Spearman, and Yvonne Harris		8. PERFORMING ORGANIZATION REPORT NO.
9. PERFORMING ORGANIZATION NAME AND ADDRESS Foster Associates, Inc. 1101 Seveteenth Street, N.W. Seventeenth Washington, D.C. 20036		10. PROGRAM ELEMENT NO. 11. CONTRACT/GRANT NO. 68-02-2592
12. SPONSORING AGENCY NAME AND ADDRESS EPA, Office of Air Quality Planning and Standards Pollutant Assessment Branch Research Triangle Park, NC 27711		13. TYPE OF REPORT AND PERIOD COVERED Final 14. SPONSORING AGENCY CODE U.S. EPA

15. SUPPLEMENTARY NOTES

Project Officer - Rayburn Morrison, OAQPS/SASD, MD-12

16. ABSTRACT

Coal analysis data and power plant continuous monitoring data were gathered, reviewed and analyzed to assess the impact of fuel coal characteristics on compliance strategies and emission regulations. Coal analysis data, on a raw and washed basis, were analyzed by individual mine, composite coal seams and USBM Producing Districts. The results indicated that composite coal seam or Producing District data cannot be used to accurately predict sulfur variabilities for individual mines. Analyses indicated that the heat content (Btu/lb) was best approximated by the normal distribution, which the sulfur content and pounds sulfur/MMBtu were best represented by the inverted gamma distribution which was slightly superior to the lognormal distribution. Analysis of available continuous monitoring data supported the inverse relationship between coal sulfur variability and lot size, i.e., significant reductions in relative variability of emissions occur as the averaging time increases. The continuous monitoring data indicate that while FGD systems reduce mean emission levels, the relative variabilities of outlet SO_2 concentrations are substantially greater than those of inlet SO_2 concentrations. The various analyses of coal sulfur variability identified no reliable method for coal suppliers or sonsumers to predict variability which may be critical for compliance by some coal-fired boilers to existing sulfur emission-limiting regulations.

17. KEY WORDS AND DOCUMENT ANALYSIS

a. DESCRIPTORS	b. IDENTIFIERS/OPEN ENDED TERMS	c. COSATI Field/Group
SO_2 Emissions Coal Air pollution Sulfur variability Power plants Fuel Standards Emission standards Coal sampling	Coal Sulfur variability Air pollution control	

18. DISTRIBUTION STATEMENT Unlimited	19. SECURITY CLASS *(This Report)* Unclassified	21. NO. OF PAGES
	20. SECURITY CLASS *(This page)* Unclassified	22. PRICE

ATTENTION

AS NOTED IN THE NTIS ANNOUNCEMENT, PORTIONS OF THIS REPORT ARE NOT LEGIBLE. HOWEVER, IT IS THE BEST REPRODUCTION AVAILABLE FROM THE COPY SENT TO NTIS.

United States
Environmental Protection
Agency

Office of Air Quality
Planning and Standards
Research Triangle Park NC 27711

EPA-450/5-80-008a
May 1980

PB81-111585

Air

EPA

A Statistical Study of Coal Sulfur Variability and Related Factors

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U. S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

EPA-450/5-80-008a

A Statistical Study of Coal Sulfur Variability and Related Factors

by

George R. Warholic, John E. Morton, Yimin Ngan,
James E. Spearman, and Yvonne Harris

Foster Associates, Inc.
1101 Seventeenth Street, N.W.
Washington, D.C. 20036

Contract No. 68-02-2592

EPA Project Officer: Rayburn Morrison

Prepared for

U.S. ENVIRONMENTAL PROTECTION AGENCY
Office of Air, Noise, and Radiation
Office of Air Quality Planning and Standards
Research Triangle Park, North Carolina 27711

May 1980

This report is issued by the Environmental Protection Agency to report technical data of interest to a limited number of readers. Copies are available free of charge to Federal employees, current contractors and grantees, and nonprofit organizations - in limited quantities - from the Library Services Office (MD35), Research Triangle Park, North Carolina 27711; or, for a fee, from the National Technical Information Service, 5285 Port Royal Road, Springfield, Virginia 22161.

This report was furnished to the Environmental Protection Agency by Foster Associates, Inc., Washington, D.C. in fulfillment of Contract Nos. 68-02-2592 and 68-01-5845. The contents of this report are reproduced herein as received from Foster Associates, Inc. The opinions, findings, and conclusions expressed are those of the authors and not necessarily those of the Environmental Protection Agency. Mention of company or product names is not to be considered as an endorsement by the Environmental Protection Agency.

TABLE OF CONTENTS

Page

Summary vi

1.0 Introduction 1-1

2.0 Coal Sulfur Variability 2-1
2.1 Background 2-1
2.2 Types of Sulfur Dioxide Emission-Limiting Regulations 2-5
2.2.1 New Source Performance Standards (NSPS) 2-5
2.2.2 State Implementation Plans (SIPs). . 2-7
2.2.3 Impact of the Length of the Averaging Time Interval on Compliance with Emission Regulations . . . 2-8
2.2.4 Impact of the Averaging Time Frame and Allowable Exceedances on Compliance with Emissions Regulations . . 2-12
2.3 Conversion of Coal Sulfur to Sulfur Dioxide 2-22
2.4 Effect of the Choice of Statistical Distribution on Compliance with Sulfur Dioxide Emission Regulations 2-24
2.5 Theoretical Effects of Measurement Error on the Relative Standard Deviation of Lbs S/MMBtu 2-30
2.5.1 Estimates of Sampling and Analytical Error 2-31
2.5.2 Analysis of the Impact of Measurement Error on the RSD of Lbs S/MMBtu 2-33

3.0 Methodology 3-1
3.1 Data Collection 3-1
3.2 Data Base 3-5
3.2.1 Coal Analysis Data File 3-5
3.2.2 Stack Monitoring Data File 3-5
3.2.3 Index to Mine Locations and Seams Produced 3-6
3.2.4 Maps of Mine Locations by Producing District, State, and County 3-7
3.3 Analysis of Data 3-8
3.3.1 Plot of Variable Vs. Time 3-9
3.3.2 Sample Statistics 3-10
3.3.3 Frequency Distribution of Observed Data 3-10
3.3.4 Comparison of Observed Distribution to Expected Distribution 3-10
3.3.5 Goodness of Fit Between the Observed and Expected Frequency Distributions 3-12

Page

4.0 Analysis of Coal Data Received from Respondents 4-1
    4.1 Sample Statistics 4-2
        4.1.1 Individual Data Sets 4-3
            4.1.1.1 RSD Versus Lot-Size 4-4
            4.1.1.2 Multiple Data Sets for Individual Mines 4-7
            4.1.1.3 Mine 10006 4-9
        4.1.2 Lot-Size Interval Analysis of Data . 4-11
            4.1.2.1 Theoretical Relationship Between RSD of Lbs S/MMBtu and Lot-Size . . . 4-12
            4.1.2.2 Observed Relationship Between RSD of Lbs S/MMBtu and Lot-Size . . . 4-16
        4.1.3 Analysis of Data on an Aggregate Basis 4-21
    4.2 Predictive Ability of Producing District and Seam Data for Individual Mines 4-23
        4.2.1 Methodology 4-24
        4.2.2 Analysis of Predictive Capabilities of Seam Data 4-24
        4.2.3 Analysis of the Predictive Capabilities of Producing District Data 4-27
    4.3 Analysis of the Statistical Distributions of Coal Characteristics 4-27
        4.3.1 Methods of Analysis 4-29
        4.3.2 Results of Best Fit Analysis 4-33

5.0 Analysis of Bureau of Mines Data 5-1
    5.1 Analysis of Selected Coal Seams 5-2
        5.1.1 Northern Appalachian 5-2
        5.1.2 Southern Appalachian 5-4
        5.1.3 Mid-Continent 5-4
        5.1.4 Western 5-4
        5.1.5 Comparison of Analysis of Seam Data from Bureau of Mines Data with Data Received from Respondents 5-5
    5.2 Analysis of Selected Mines 5-5
        5.2.1 Mine 01200, Upper Freeport Seam . . 5-7
        5.2.2 Mine 00950, Pittsburgh Seam 5-7
        5.2.3 Mine 07290, Middle Kittanning Seam . 5-7
        5.2.4 Mine 00614, Various Seams 5-10
        5.2.5 Mine 00637, Upper Elkhorn #3 and Hazard #4 Seams 5-10
        5.2.6 Mine 02557, Upper Elkhorn #3 Seam . 5-10
        5.2.7 Mines 04184 and 04204, Hazard #5-A Seam 5-11

Page

5.2.8 Mine 04978, Herrin #6 Seam . . . 5-11
5.2.9 Mine 07310, Morris #2 Seam . . . 5-12
5.2.10 Mine 04730, Indiana Seams #5, #6, and #7 5-12
5.2.11 Mines 00305 and 07243, Black Creek Seam 5-12
5.2.12 Mine 00750, Various Seams . . . 5-13
5.2.13 Mine 02020, F Seam 5-13
5.2.14 Mine 07235, Hiawatha Seam . . . 5-13
5.2.15 Summary of Analysis of Individual Mine Data from "Detail Tape" 5-14

6.0 Analysis of Continuous Monitoring Data for Sulfur Dioxide Emissions 6-1
6.1 Description of Data 6-1
6.2 Analysis of Data 6-2
6.3 Implications of Emissions Analysis 6-6

7.0 Coal Sulfur Regression Analysis 7-1
7.1 Objectives 7-1
7.2 Background 7-1
7.3 Average Sulfur Content Regression Results . 7-5
7.3.1 Producing District 10 7-6
7.3.2 Producing District 8 7-10
7.3.3 Producing District 4 7-14
7.3.4 Mine #3 7-16
7.3.5 Summary of Average Sulfur Content Regression Results 7-19
7.4 Sulfur Variability Regression Results . . . 7-22
7.4.1 Relative Standard Deviation Regressions 7-22
7.4.2 Variance Regressions 7-26
7.4.3 Summary of Sulfur Variability Regression Results 7-28

8.0 Conclusions and Recommendations 8-1
8.1 Conclusions 8-1
8.2 Recommendations 8-5

Appendix A: Derivation of Equations Used for the Development of Tables 10, 11, and 12 . . A-1
Appendix B: Derivation of Formula for True and Measured RSD B-1
Appendix C: Coal Sulfur Analyses Data Base Format . C-1
Appendix D: Stack Monitoring Data Base Format . . . D-1
Appendix E: Index to Mine Locations and Seams Produced E-1
Appendix F: Maps of Mine Locations by Producing District, State and County F-1

Page

Appendix G: Sample Output of Analytical Program . . G-1
Appendix H: Summary of Salient Characteristics of Data Sets Analyzed H-1
Appendix J: Analysis of Data on an Aggregate Basis by Producing District J-1
Appendix K: Analysis of the Statistical Distributions of Coal Characteristics K-1

SUMMARY

Coal analysis data and limited continuous monitoring data were obtained from various electric utilities, coal companies, Government agencies, and other organizations. These data were documented, edited, and placed into databases for the analysis of coal sulfur variability. Statistical programs were used to analyze coal analyses as well as continuous monitoring data to assist EPA in its assessment of the impacts of coal characteristics on compliance strategies and emissions regulations.

Although the data gathered and analyzed in this study appear to be the best currently available, it should be noted that the data have certain limitations for use in statistical analysis. In particular, the coal data were not the result of controlled experiments, but rather historical data, generally used for establishing coal prices and monitoring overall coal quality. In most instances the sources of data reported whether ASTM sampling and analysis procedures were used. However, it could not be ascertained how rigorously ASTM procedures were followed. Finally, it was not possible to isolate the effects of factors such as mining techniques and coal handling operations.

Coal analysis data, on a raw and washed basis, were analyzed by individual mine, composite coal seams, and USBM Producing Districts. Variables analyzed included volume (tons), heat content (Btu/lb), sulfur content (weight percent) and pounds of sulfur per million Btu (lbs S/MMBtu). The sample statistics from these analyses failed to identify any consistent, predictable relationships which would explain coal sulfur variabilities. This study concluded that composite coal seam or Producing District data cannot be used

to accurately predict sulfur variabilities for individual mines.

The statistical analyses conducted in this study were based on a simple model assuming independent variance. More sophisticated autocorrelative models were not investigated. However, given the limitations of available data, it is questionable whether such models would yield better results. Visual examination of the time plots used in this study suggests that the data sets contain little, if any, autocorrelation. Nevertheless, these preliminary findings should not preclude the investigation of autocorrelative models in future studies.

Results of a simulation study, which assumed independence, indicated that theoretically coal sulfur variability should decrease with increasing lot-sizes. However, in contrast to the theoretical results and some previous studies, the coal data analyzed by sorting the data sets by lot-size groupings and comparing the variabilities between lot-sizes failed to provide strong support for an inverse relationship at any level of aggregation. These results may be due, in part, to the lack of statistical control.

Various regression analyses performed by mine, seam, and Producing District failed to provide any good explanatory variables for coal sulfur variability. The results of these analyses tend to support the hypothesis that the primary factors affecting coal sulfur distributions are geologic factors, mining techniques, and coal handling procedures, while chemical and physical properties of coal are secondary factors.

Comparisons of the observed frequency distributions to the normal, lognormal, and inverted gamma distributions indicated that coal heat contents are best approximated by the normal distribution, while sulfur contents and lbs S/MMBtu are best represented by the inverted gamma which appears to be slightly superior to the lognormal distribution.

Analysis of the limited continuous monitoring data indicated that significant reductions in the relative variability of emissions can be achieved by increasing the averaging time interval from one-hour, to three-hours, to 24-hours, to 30-days. These findings support the theoretical, inverse relationship between coal sulfur variability and lot-size, since increasing the averaging interval is equivalent to increasing the lot-size of coal burned. However, it should be noted that the reductions in relative variability were less than would be expected based on statistical independence.

The analysis of continuous monitoring data also indicated that, while flue gas desulfurization (FGD) units reduce the mean level of emissions, the relative variabilities of FGD outlet SO_2 concentrations are substantially greater than the FGD inlet SO_2 concentrations.

The various analyses of coal sulfur variability identified no reliable method for coal suppliers or consumers to predict variability, which is often critical for compliance with the existing sulfur emission-limiting regulations. Coal sulfur variability is especially critical in the case of small coal-fired boilers and regulations which stipulate short averaging periods. This suggests that the language of many current regulations is not consistent with the state of knowledge concerning coal sulfur variability.

Included in this report are the results of related studies which examine the theoretical impacts of measurement error, choice of frequency distribution, and emission regulations which require a very low probability of excess emissions. Included in the Appendices of this report are summaries of the sample statistics at various levels of aggregation, sample output of the analytical programs, and expanded discussions of several topics examined in the report.

Within the overall objective of understanding the problem of coal sulfur variability and how coal producers, coal consumers, and pollution control agencies can cope with this problem, this study failed to provide simple explanations or solutions. Instead, the results illustrated the complexities of the problems and indicated the need for additional studies.

The data base and analyses in this report are not viewed as exhaustive, but rather serve to establish a base from which further studies can build in order to provide the inputs necessary to understand the consequences of sulfur variability *vis-a-vis* current sulfur dioxide emission regulations. Ideally, future studies will develop explanatory relationships which can be used in a comprehensive model to assess the impact on air quality, given the parameters for coal characteristics, mining and handling methods, combustion and control equipment, meteorological data, and other variables. Alternatively, these studies may provide data to develop new regulations which would mitigate the impact of coal sulfur variability yet achieve the objectives of existing sulfur dioxide regulations.

1.0 Introduction

The purpose of this report is to assist EPA in its analysis of the impact of coal sulfur content variability on the ability of both utility and industrial boilers to comply with the sulfur emission regulations of the current State Implementation Plans (SIP) and the existing Federal New Source Performance Standards (NSPS).

Special objectives of this study were to:

(1) Collect and consolidate all existing data pertaining to the variability of coal sulfur contents and enter these data into a computer data base.

(2) Classify and document each data set collected with respect to those factors which may influence variability in sulfur content.

(3) Physically locate the sources of the coal data sets on a map of the United States.

(4) Use the data base to prepare a report providing information on coal sulfur variability for low, medium, and high sulfur coals throughout the United States.

(5) Analyze the impact of coal sulfur variability on the ability of coal-fired boilers to comply with sulfur emission regulations.

Data were obtained from coal companies, electric utilities, EPA files, Bureau of Mines, and previous studies of coal sulfur variability.

This study is basically an extension of an earlier EPA study[1] concerning coal sulfur variability which focused primarily on low-sulfur coals. This study examines the findings of the earlier study through the use of additional statistical techniques and a more extensive data base for low-sulfur coals as well as medium- and high-sulfur coals.

In recent years air pollution control agencies and facilities affected by SIP's and NSPS have been increasingly aware of the problems associated with the impact of coal sulfur variability on meeting sulfur emission regulations which specify an emission ceiling never to be exceeded during short-term averaging periods such as one hour, three hours, 24 hours or one month. Germane to the understanding of this problem is an examination of how the short-term sulfur emissions from a coal relate to the nominal or long-term average sulfur and heat contents.

Coal is not a homogeneous commodity and is subject to variations in physical characteristics. The degree of variation in the sulfur and heat contents has a substantial impact on the ability of a coal to comply with sulfur emission regulations. This report attempts to quantify these variations for various coal mines, seams, and producing areas and to identify those factors which may contribute to these variations.

Some insight into the impact of coal sulfur variability can be gained from a review of the results from an EPA-sponsored study on Louisville Gas and Electric Company's

---

1/ PEDCo Environmental, "Preliminary Evaluation of Sulfur Variability in Low-Sulfur Coals From Selected Mines", EPA Publication No. EPA-450/3-77-044, July 1977.

Cane Run Unit No. 4.[1] The results of this study exhibited an average emission rate of 0.95 lbs SO_2/MMBtu for the entire test period, during August-December, 1977. However, because of the inherent coal sulfur variability and variability associated with the desulfurization processes there was a substantial variation of emissions about the mean.

An analysis of the data indicated that the 24-hour averages were lognormally distributed. The geometric means, geometric standard deviations, and implied exceedance rates are set out in Table 1. From Table 1 the reduction in variability is readily apparent from the declining geometric standard deviations and exceedance rates as the averaging interval, and consequently the volume or lot-size of coal burned, is increased.

TABLE 1

COMPARISON OF GEOMETRIC MEANS, GEOMETRIC STANDARD DEVIATIONS, AND IMPLIED EXCEEDANCE RATES FOR SELECTED AVERAGING INTERVALS

	Averaging Interval				
	3-Hour	24-Hour	7-Day	14-Day	30-Day
No. of Observations	678	89	12	6	3
Geometric Mean	0.885	0.908	0.95	0.95	0.945
Geometric Standard Deviation	1.462	1.352	1.26	1.17	1.111
Exceedance Rate (Percent)	21	18	16	7	1

---

[1] Based on the results reported in the publication "Air Pollution Emission Test", Vol. I, as well as additional analyses by the Energy Strategies Branch of the Environmental Protection Agency. USEPA, Office of Air Quality Planning and Standards, Emission Measurement Branch, "Air Pollution Emmission Test, Volume I, First Interim Report: Continuous Sulfur Dioxide Monitoring at Steam Generators", EMB Report No. 77SPP23A, August, 1978.

2.0 Coal Sulfur Variability

2.1 Background

The initial New Source Performance Standards (NSPS) applicable to SO_2 emissions from fossil fuel-fired steam generating units of more than 250 MMBtu/hr heat input prohibit SO_2 emissions in excess of 1.2 lbs/MMBtu when solid fossil fuel is burned.[1/] This standard was promulgated on December 23, 1971. Since 1971 several studies have indicated that the definition of NSPS "complying coal" required to meet this standard is more complex than was originally envisioned in the background information for the standard due to the variability of sulfur and heat contents of coal burned during short time periods, e.g., one hour or 24 hours. Based on available data, it appears that much of the previously identified "complying coal" would result in excess SO_2 emissions when burned.

In addition to the NSPS, many State Implementation Plans (SIPs) also specify a sulfur or sulfur dioxide emissions ceiling for coal-fired generating units. Although in many cases these regulations are less stringent than the NSPS, the definition of a complying coal, based on the coal sulfur content, is equally complex.

The basic problem in defining a compliance coal is that coal, even in a narrowly defined producing area, is not a homogeneous commodity, but is subject to variations in physical characteristics. The variations in coal sulfur and heat

---

[1/] Applicable to plants for which construction commenced between August 17, 1971 and September 18, 1978. Plants for which construction commenced after September 18, 1978 are subject to a revised NSPS.

contents jointly contribute to variations in the emissions (pounds of SO_2/MMBtu or pounds of S/MMBtu) resulting from coal combustion.

Sulfur in coal occurs in three forms: organic, sulfate, and pyritic. Organic sulfur is an integral part of the coal and generally cannot be removed by existing coal cleaning techniques. Organic sulfur generally comprises about 30 to 70 percent of the total sulfur content of most coals. Sulfur in the sulfate form is a water soluble oxidation product formed from the weathering of the iron sulfide in coal and can be readily removed through coal cleaning. Sulfate sulfur contents are usually less then 0.05 percent. Pyritic sulfur occurs in coal as pyrite and/or marcasite, which are iron sulfides. Pyrite is a relatively heavy mineral with a specific gravity of 5.0 compared to coal which has a specific gravity of 1.7 or less. The pyrite content of most coals can be significantly reduced through current coal preparation processes.

The total sulfur content (organic, sulfate, and pyritic) of coals in the United States generally ranges from about 0.2 percent to 7.0 percent. The total sulfur content, as well as the ratio of organic to inorganic sulfur, varies widely among coal seams, geographical locations, and frequently among mines operating in the same coal seam in the same geographical location. These natural variations in sulfur content, as well as variations in other physical characteristics, are attributed to many factors which include:

1. Mode of accumulation and burial of coal-forming vegetal matter.

2. Structure and chemical composition of the coal forming vegetal matter.

3. Extent of inert material washed into the coal swamp at the time of accumulation.

4. Age of the coal deposit and its geographical location.

5. Subsequent geologic history of the deposit, such as permeation of ground water.

The variability of sulfur contents, as observed in coal analyses, are a result of the above natural events plus numerous other factors resulting from various mining, processing, transportation, sampling, and utilization techniques.

A summary of the pertinent factors which have been identified as potential sources of coal sulfur variability is set out in Table 2. Since many of these factors are interrelated, a study of coal sulfur variability requires well documented data in order to isolate and individually examine those factors which may contribute to variability. A detailed analysis of each of these factors is not possible due to the large number of factors and their interdependence and the lack of adequate data.

Although many electric utilities, coal companies, and research organizations have suitable data, the cost of assembling and analyzing these data in many cases is prohibitive. In addition, several companies reported that they have assembled and/or analyzed coal sulfur variability data but the information is considered proprietary due to pending legal actions or company policies.

TABLE 2

PERTINENT FACTORS IN STUDYING SULFUR VARIABILITY

1. Type of coal

   Organic and inorganic sulfur content.
   Distribution of sulfur in coal (coarse pyrite or finely disseminated throughout the coal). The form in which the sulfur occurs is significant when the coal is washed.

2. Stage of sampling

   Core drilling (or channel samples after operation).
   Run-of-mine production.
   After preparation, cleaning.
   As received at utility plant/consumer.
   As burned.

3. Coal blending and processing

4. Mining plan (selective)

5. Mining technique

   Number and location of machines, type of mining

6. Location of coal

   Seam.
   Mine.
   Region or district.

7. Averaging times/tonnages

   Daily.
   Weekly.
   Monthly.
   Other.

8. Sampling procedure

   Amount of coal sampled.
   Method of collecting increments for gross sample.
   Sample variation.

9. Analytical method

10. Cleaning technique

Source: PEDCo Environmental, "Preliminary Evaluation of Sulfur Variability in Low-Sulfur Coals for Selected Mines", EPA Publication No. EPA-450/3-77-044, July 1977.

The impact of the variations in the physical characteristics of coal is largely dependent upon the specific requirements of a given regulation. One of the most pertinent factors concerns the specified or implied averaging period associated with the regulation. Previous studies based on statistical theory indicate that coal variability increases when coal samples and analyses are based on successively smaller volumes of coal or shorter averaging periods. However, studies of individual data sets have produced inconsistent results.

2.2 Types of Sulfur Dioxide Emission-Limiting Regulations

A review of the existing sulfur dioxide emission-limiting regulations for the electric utility industry clearly shows the complexity and diversity of the regulations applicable to coal-fired generating units. These regulations vary with respect to the units of measure in which the limitation is expressed as well as the scope of equipment (boiler, stack, or entire plant) to which the regulations apply. In addition, the regulations may or may not specify an averaging time. When averaging periods are specified in the regulations, they vary among the individual regulations, while unspecified averaging times have generally resulted in some assumed averaging time by the plant operators or the pollution control agencies. The following discussion examines the diversity of sulfur dioxide emission-limiting regulations and provides some perspective as to the number of plants and annual coal requirements controlled by specific types of regulations.

2.2.1 New Source Performance Standards (NSPS)

The NSPS limits the emissions of sulfur dioxide (SO_2) to 1.2 lbs SO_2/MMBtu of heat input. This standard applies to all coal-fired steam-generating units rated at more than 250 MMBtu/hr heat input. Although the NSPS specifies test methods for determining excess emissions, the NSPS is ambiguous with respect to the averaging time required for compliance with the 1.2 lbs SO_2/MMBtu limit and has been subject to a variety of interpretations.

A survey conducted in early 1978 identified 23 coal-fired electric utility generating units operating under the NSPS.1/ Based on a total installed capacity of 9,078 MW, these 23 units would require approximately 32 million tons of coal per year.2/

The survey revealed that a variety of averaging intervals were being used for reporting SO_2 emissions. A summary of the reported averaging intervals is set out in Table 3.

TABLE 3

SUMMARY OF AVERAGING TIMES FOR SO_2 EMISSIONS
AS REPORTED BY NSPS UNITS

Averaging Interval	Number of Units
24-hour	1
3-hour	4
3-hour moving	4
2-hour	1
1-hour	2
<1-hour	1
Not determined	10
Total	23

Source: Foster Associates, Inc.

1/ "Analysis of Currently Operating Coal-Fired Power Units Subject to the Environmental Protection Agency's New Source Performance Standards", prepared for EPA by Foster Associates, Inc., February 24, 1978 (unpublished report).

2/ Estimated based on 0.4 tons of coal/MWH.

2.2.2 State Implementation Plans (SIPs)

A similar survey of the distribution and applicability of the various SIP sulfur dioxide emission-limiting regulations was conducted by EPA.[1] This study identified 373 power plants operating under SIP sulfur regulations. The annual coal consumption of these plants was estimated at 446 million tons.

As shown in Table 4, the regulations vary with respect to the units of measure in which the limitations are expressed. The regulations are most frequently expressed in terms of lbs SO_2/MMBtu which account for 61.2 percent of annual coal consumption. Regulations limiting the percent sulfur of the coal and the lbs SO_2/hr control the second largest amount of coal at 10.1 percent each. Limitations expressed in ppm of SO_2, and lbs S/MMBtu occur less frequently and control a total of about 13.4 percent of annual coal consumption. In addition, the SIPs applicable to 28 plants, which account for approximately 5.2 percent of annual coal consumption, specify no limit for sulfur dioxide emissions.

---

[1] "The Types of SIP SO_2 Emission-Limiting Regulations: Their Distribution and Applicability", Memorandum from R. D. Bauman, Chief, Energy Strategies Branch to W. C. Barber, Director, Office of Air Quality Planning and Standards, November 3, 1978.

TABLE 4

SUMMARY OF SIP SULFUR DIOXIDE CONTROL REGULATIONS

Type of Regulation	Amount Controlled		
	Number of Plants	Annual Coal Consumption (Million Tons)	Annual Coal Consumption (Percent)
Lbs SO_2/MMBtu	215	273	61.2
Lbs S/MMBtu	19	15	3.4
Percent Sulfur	54	45	10.1
Lbs SO_2/hr	23	45	10.1
ppm SO_2			
- at stack (exhaust gas)	3	6	1.3
- at ground (AAQ)	31	39	8.7
No emission limit	28	23	5.2
Total	373	446	100.0

Source: EPA.

The previous discussion has focused on electric utility plants due to the lack of publicly available data for the industrial sector. Preliminary data indicate that coal consumption in the industrial sector during 1978 was approximately 59 million tons.[1] Although coal consumption in the industrial sector is substantially less than in the utility sector, most large industrial plants are subject to sulfur dioxide emissions-limiting regulations of the NSPS and SIPs. However, available data do not permit a tabulation of the number of plants and annual volumes of coal controlled by specific regulations.

2.2.3 Impact of the Length of the Averaging Time Interval on Compliance with Emission Regulations

Some insight into the impact of the length of the averaging time interval can be gained from an examination of the results of the EPA-sponsored study on the Cane Run Unit

---

1/ DOE, Weekly Coal Report, No. 77, March 23, 1979.

No. 4, noted in the introduction of this report. Cane Run Unit No. 4, operated by Louisville Gas and Electric Company, has a rated capacity of 178 MW of electricity and is equipped with a flue gas treatment system designed to achieve an 85 percent reduction of sulfur dioxide emissions, in order to comply with a SIP regulation of 1.2 lbs SO_2/MMBtu. The plant is an intermediate-load facility which tracks the system load. Estimated coal consumption for Unit No. 4 assuming operation at full capacity as well as at a 57 percent capacity factor, at which the unit operated in 1976, is set out in Table 5.

TABLE 5

ESTIMATED COAL CONSUMPTION, CANE RUN UNIT NO. 4

	Volume (Tons)	
Interval	Full Capacity	57 Percent of Capacity
1-hour	75	43
3-hour	225	128
24-hour	1,800	1,026
30-day	54,000	30,780
1-year	657,000	374,490

The nominal properties of the raw, Western Kentucky coal burned by Cane Run Unit No. 4 are: 3.7 percent sulfur; 11,200 Btu/lb; and 12.6 percent ash.

The results of this study exhibited an average emission rate of 0.95 lbs SO_2/MMBtu for the entire test period, during August-December, 1977. However, because of the inherent coal sulfur variability and variability associated with the desulfurization processes there was a substantial variation of emissions about the mean. Set out in Table 6 are the minimum and maximum values of emissions as well as the rates at which emissions exceeded a limit of 1.2 lbs SO_2/MMBtu for 3-hour and 24-hour averaging intervals.

TABLE 6

MAXIMUM AND MINIMUM EMISSIONS AND EXCEEDANCES CANE RUN UNIT NO. 4

Averaging Interval	Emissions, lb SO_2/MMBtu Minimum	Emissions, lb SO_2/MMBtu Maximum	No. of Exceedances	Percentage of Observations Exceeding, 1.2 lb SO_2/MMBtu
3-hour	0.19	2.45	184	24
24-hour	0.37	1.74	18	20

An analysis of the data indicated that the 24-hour averages were lognormally distributed. The geometric means and geometric standard deviations for various averaging intervals are set out in Table 7. The implied exceedance rates, calculated through the use of normal distribution theory for logarithms, are also set out in Table 7.

TABLE 7

COMPARISON OF GEOMETRIC MEANS, GEOMETRIC STANDARD DEVIATIONS, AND IMPLIED EXCEEDANCE RATES FOR SELECTED AVERAGING INTERVALS

	Averaging Interval				
	3-Hour	24-Hour	7-Day	14-Day	30-Day
No. of Observations	678	89	12	6	3
Geometric Mean	0.885	0.908	0.95	0.95	0.945
Geometric Standard Deviation	1.462	1.352	1.26	1.17	1.111
Exceedance Rate (Percent)	21	18	16	7	1

From Table 7 the reduction in the exceedance rate is readily apparent as the averaging interval, and consequently the volume of coal burned, is increased.

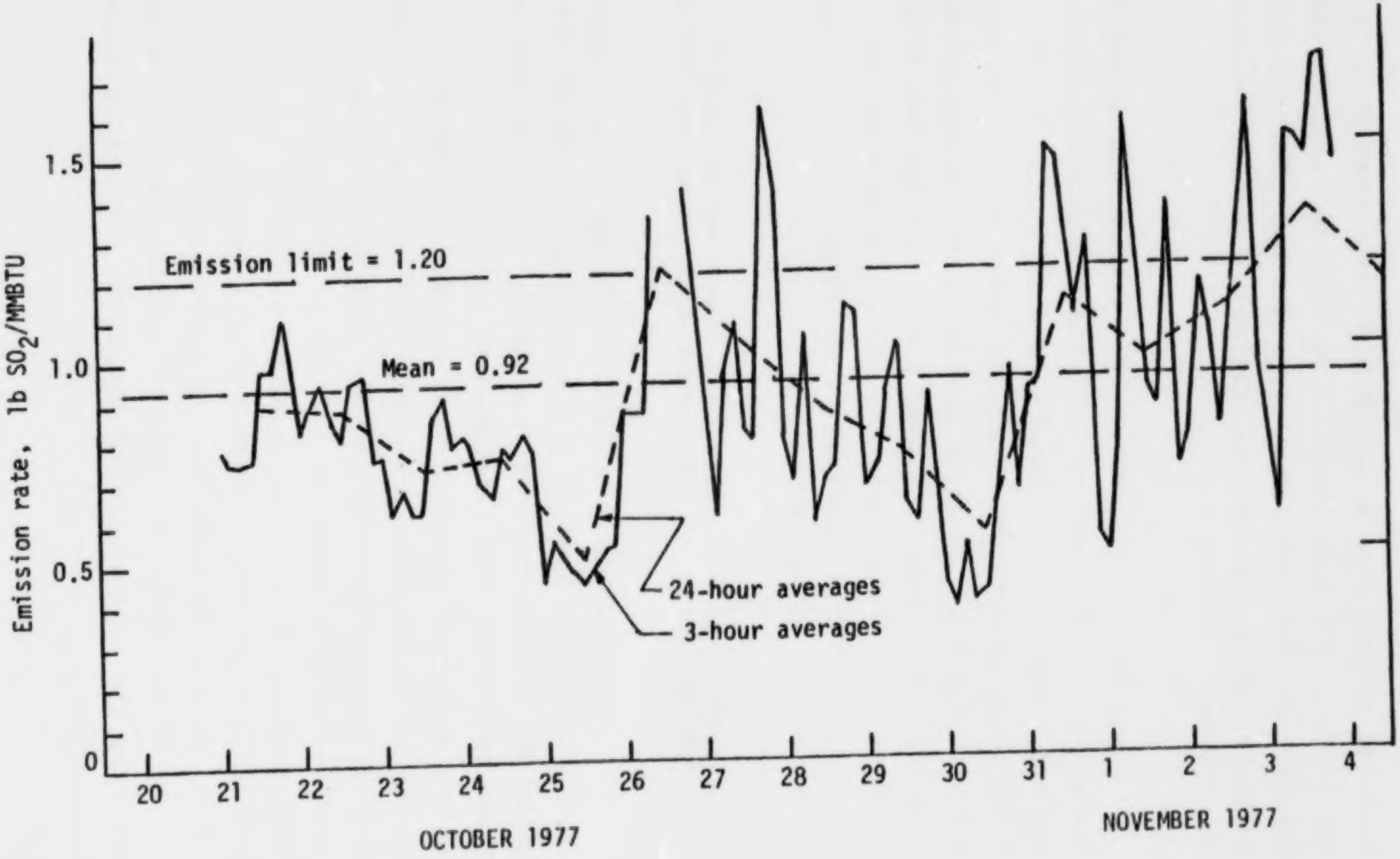
Figure 1 provides a graphical display of the variability of emission rates over successive 3-hour and 24-hour intervals for the period October 21 to November 4, 1977. Emissions for this 15-day period averaged 0.92 lbs SO_2/MMBtu. The 3-hour

FIGURE 1

EMISSIONS FROM FLUE GAS DESULFURIZATION UNIT

Louisville Gas and Electric's Cane Run Unit No. 4

High-sulfur coal; maximum sustainable output = 178 MW



Source: Based on data from "Air Pollution Emission Test", Vol. 1, analyzed by Energy Strategies Branch, Office of Air quality Planning and Standards, USEPA.

averages ranged from a low of 0.38 to a high of 1.71, while the 24-hour averages ranged from 0.50 to 1.34. Based on a standard of 1.2, the 3-hour averages exceeded the standard 20 times, while the 24-hour averages exceeded the standard two times.

The Cane Run Unit No. 4 data are certainly not representative of all coals or all flue gas desulfurization systems. However, the data demonstrate that due to emissions variability, the averaging basis associated with an emission limit is critical to the determination of what constitutes compliance with that limit. It follows from this analysis that the smaller the amount of coal burned per unit time, the more difficult it is to comply with the standard if the averaging basis is time.

2.2.4 Impact of Averaging Time Frame and Allowable Exceedances on Compliance with Emissions Regulations

In order to avoid ambiguity, sulfur emission-limiting regulations must specify the averaging time frame for the emissions measurement and the number of exceedances permitted. These exceedance restrictions are generally stated in terms of the number of days per year or days per month for which excess emissions are permitted. For example, a sulfur dioxide emission regulation may require that the sulfur dioxide emission level is 1.2 lbs SO_2/MMBtu based on a 24-hour average never to be exceeded or not to be exceeded more than 2 days per year. Such a sulfur dioxide emission regulation forces the coal user to consider not only the probability of exceeding the 24-hour average sulfur dioxide limit but also the probability of exceeding the yearly allowable exceedances.

Suppose the probability of a source being in violation of the sulfur dioxide emission level on any particular day is known. The question then arises as to what is the expected number of days, for any specified time frame, in which the source would be in violation of the sulfur dioxide emission regulation. The binomial distribution, for example, can be used to illustrate the impact of the averaging time frame on compliance.[1/]

The binomial distribution is a probability distribution which describes independent, identically distributed random samples. The number of days in violation of a sulfur standard over a 365-day period can be considered to be binomially distributed if the probability of violation on the first day equals the probability of violation on the second or any other day. The equation which describes this distribution is as follows:

$$P(X = k) = \frac{n!}{k!\ (n - k)!}\ p^k\ (1 - p)^{n - k}$$

where $P(X = k)$ = probability of k violations in n days

p = probability of violation on any particular day

n = number of days being considered

k = number of violations being considered

$!$ = factorial

[1/] Determining the probability of a source being in violation on a particular day is a complex statistical problem. The probability is not likely to be constant and is a function of the emission level on previous days (autocorrelative process). To illustrate the implications of alternative averaging intervals a binomial distribution was assumed. However, this example is not to suggest that the process should be modeled via the binomial distribution.

A simple way to calculate these probabilities for the series $k=0$, $k=1$, etc., is to use the following pair of equations:

$$P(X = 0) = (1 - p)^n$$

$$P(X = k) = \frac{(n - k + 1)\ p}{k\ (1 - p)} \quad P(X = k - 1)$$

In this analysis these calculations were performed for $n = 30$ and $n = 365$, for $k = 0$ through $k = 6$, and for various values of p.

Figure 2 shows the frequency distribution for $n = 365$ and $p = 0.001$, $p = 0.005$, and $p = 0.020$. In the first case, the most likely number of violations is zero; in the second case, it is one; and in the third case, it is seven. The most likely number of exceedances is, by definition, the mode of the frequency distribution. The "expected exceedances" are equal to n multiplied by p. It can be shown that this quantity equals the weighted average of all the possible outcomes -- 0 exceedances, 1 exceedance, 2 exceedances, etc. -- whereby the weight assigned to a given outcome equals the probability of that outcome.[1] For example, take the situation where $p = 0.001$ and $n = 365$. As Figure 2 shows, there is a 69.41 percent chance of no exceedances, a 25.36 percent chance of one exceedance, a 4.62 percent chance of two exceedances, a 0.56 percent chance of three exceedances, and a 0.05 percent chance of four exceedances. The expected exceedances are:

$$0\ (.6941) = 1\ (1.2536) + 2\ (.0462) + 3\ (.0056) \\ + 4\ (.0005) = 0.365 = 365\ (.001)$$

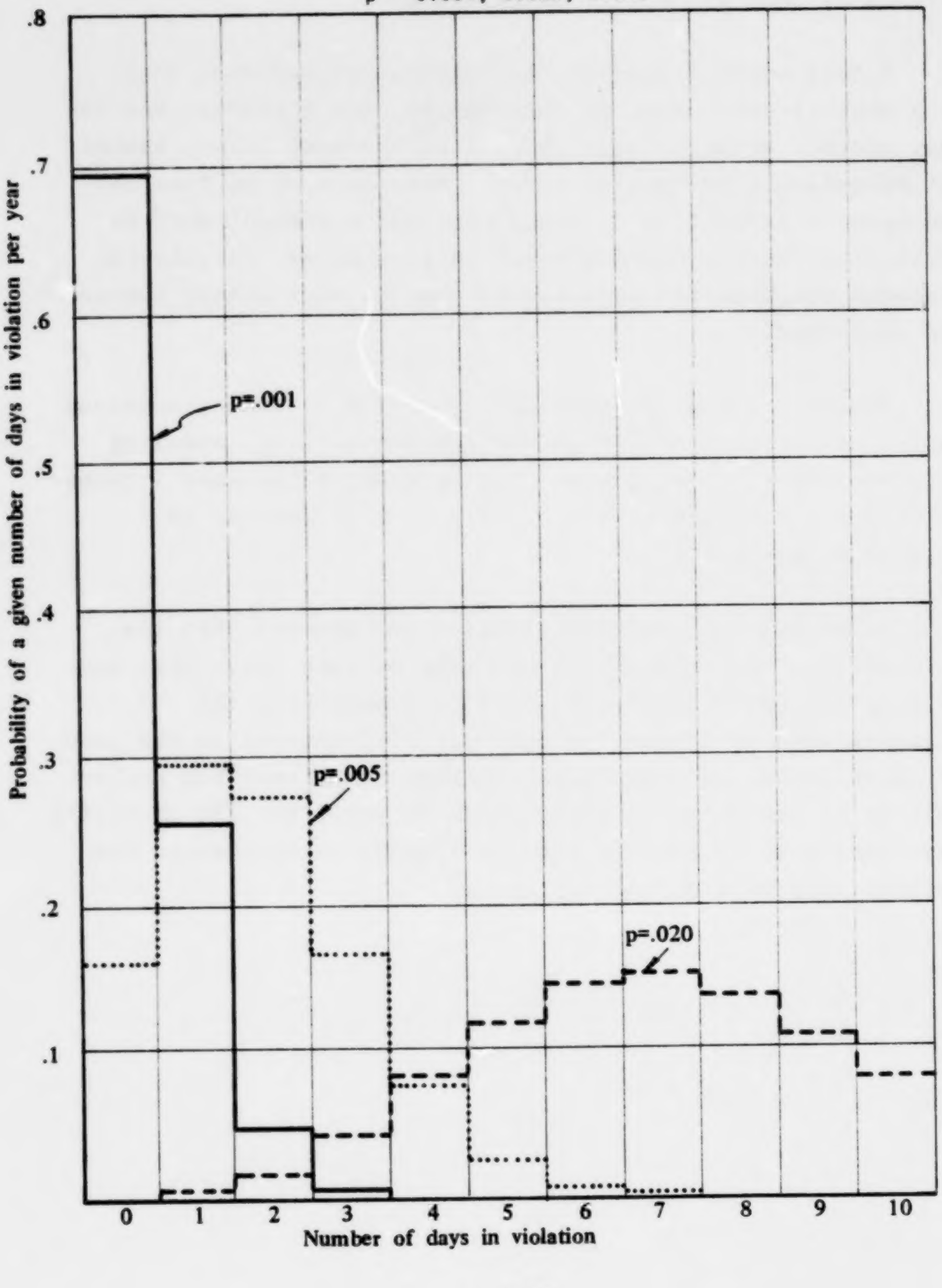
---

[1] Paul L. Meyer, Introductory Probability and Statistical Applications, 2d ed. (Reading, Mass.: Addison-Wesley, 1972), p. 120.

FIGURE 2

FREQUENCY DISTRIBUTION OF EXCEEDANCES

$n = 365$
$p = 0.001, 0.005, 0.020$



The probability of five or more exceedances is so low that it can be ignored in this calculation.

Tables 8 and 9 compare the expected exceedances with the most likely number of exceedances, for a 365-day and 30-day period, respectively. Note that the most likely number of exceedances is usually lower. Phrases such as "one-day-per-year standard" or "two-day-per-year standard" must be used with caution because these phrases do not distinguish between the expected exceedances and the most likely number of exceedances.

Figure 3 shows the probability $P(X > k)$ on the vertical axis, as a function of p on the horizontal axis, assuming $n = 365$ and $k = 3$ or $k = 6$. Figure 4 shows the same information for $k = 0$ through $k = 6$. Figure 5 is similar to Figure 4 except that $n = 30$.

Note that the binomial distribution assumes that the probability of violation is the same on each day. This condition may not be satisfied by a coal burning plant. Many factors such as changes in coal quality, changes in the load or burn ratio, and technical problems can affect the probability of violation on any particular day. The use of moving averages also introduces autocorrelation which changes the probability on a day-to-day basis.

TABLE 8

Comparison of expected exceedances with the most likely number of exceedances, over 365 days.

Probability of Exceedance on a Given Day	Expected Exceedances	Most Likely Number of Exceedances
.001	.4	0
.002	.7	0
.003	1.1	1
.004	1.5	1
.005	1.8	1
.006	2.2	2
.007	2.6	2
.008	2.9	2
.009	3.3	3
.010	3.7	3
.011	4.0	4
.012	4.4	4
.013	4.7	4
.014	5.1	5
.015	5.5	5
.016	5.8	5
.020	7.3	7

TABLE 9

Comparison of expected exceedances with the most likely number of exceedances, over 30 days.

Probability of Exceedance on a Given Day	Expected Exceedances	Most Likely Number of Exceedances
.01	.3	0
.02	.6	0
.03	.9	0
.04	1.2	1
.05	1.5	1
.06	1.8	1
.07	2.1	2
.08	2.4	2
.09	2.7	2
.10	3.0	3
.11	3.3	3
.12	3.6	3
.13	3.9	4
.14	4.2	4
.15	4.5	4

FIGURE 3

PROBABILITY OF MORE THAN X DAYS IN VIOLATION PER YEAR AS FUNCTION OF THE PROBABILITY OF VIOLATION OF AN INDIVIDUAL DAY

(n = 365, k = 3 and 6)

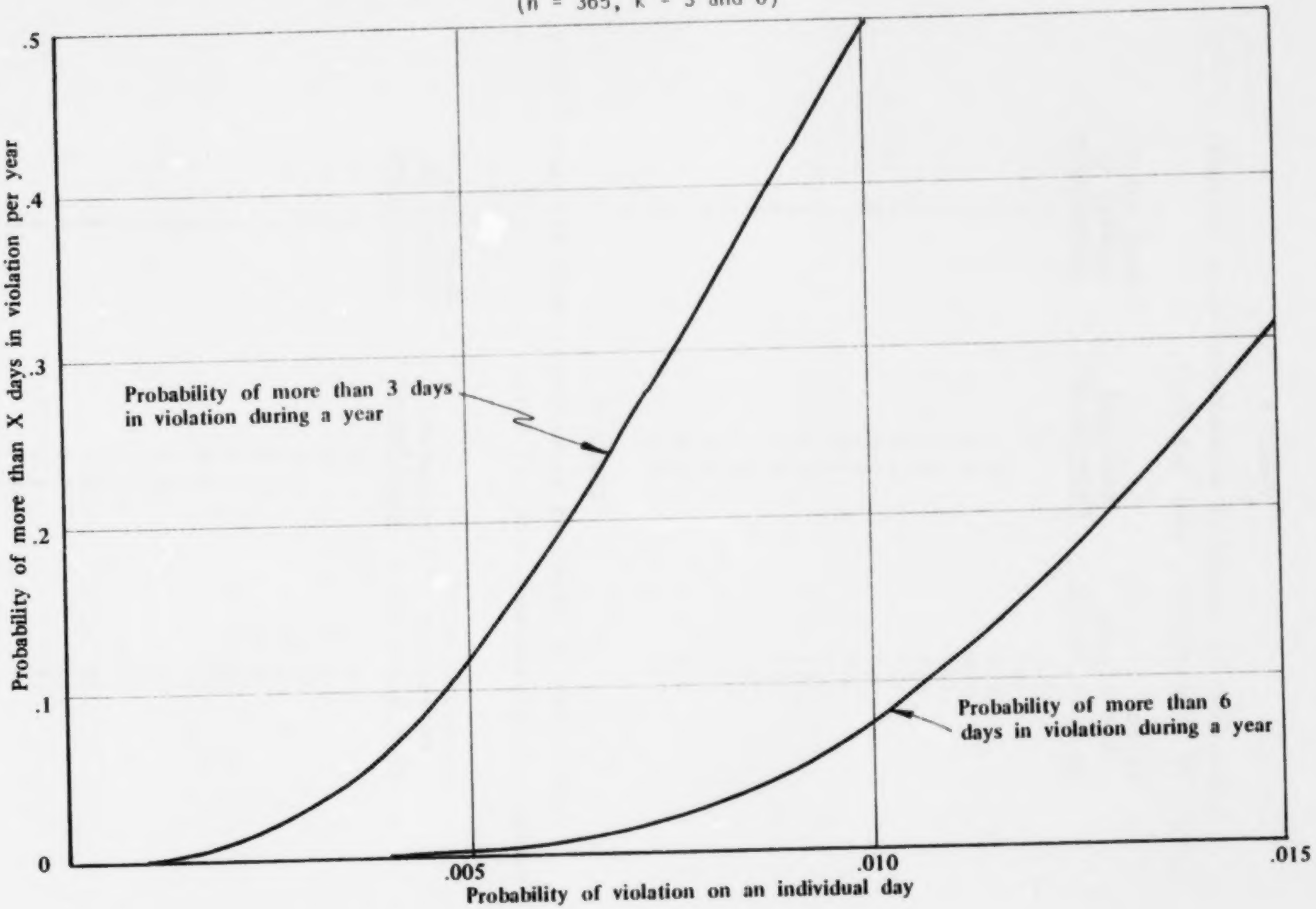


FIGURE 4

PROBABILITY OF MORE THAN X DAYS IN VIOLATION PER YEAR AS A FUNCTION OF THE PROBABILITY OF VIOLATION ON AN INDIVIDUAL DAY

(n = 365, k = 0 to 6)

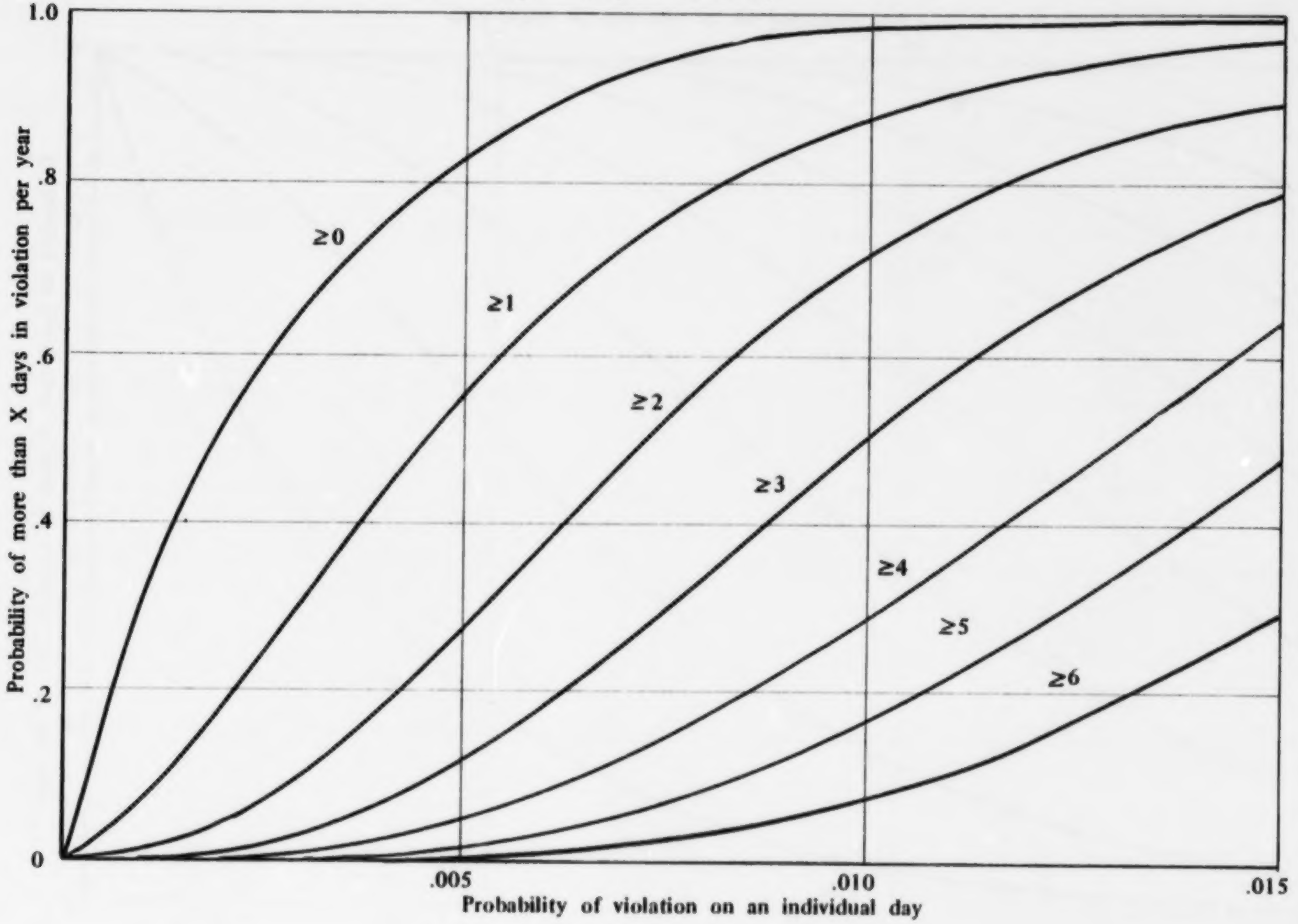
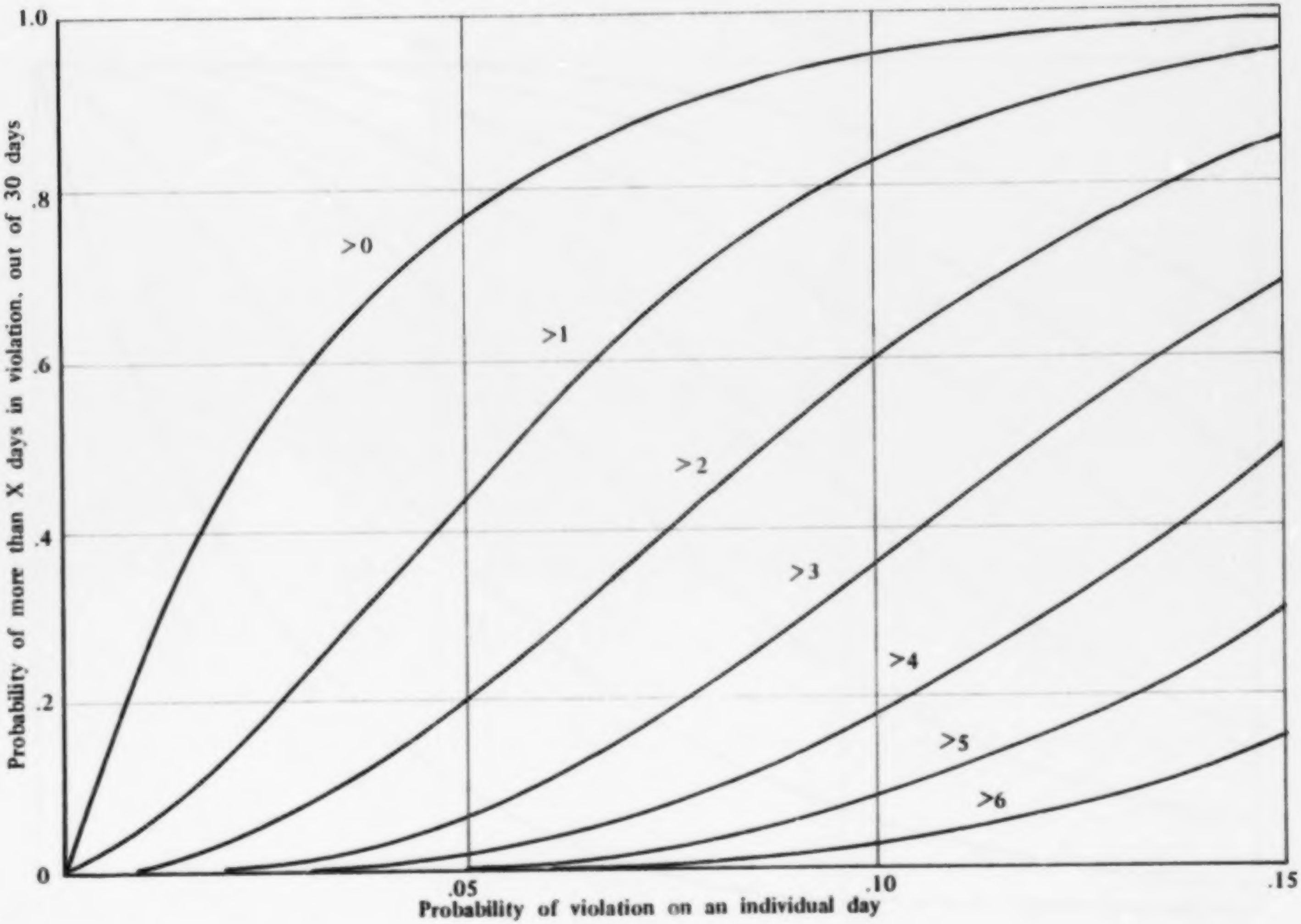


FIGURE 5

PROBABILITY OF MORE THAN X DAYS IN VIOLATION PER 30 DAYS AS A FUNCTION
OF THE PROBABILITY OF VIOLATION ON AN INDIVIDUAL DAY

(n = 30, k = 0 to 6)



The results of this analysis indicate two important statistical characteristics.

The first is that the probability of violation in any individual day must be much lower to limit the probability of exceeding a given number of violations than if one was content to simply average the same number of violations. For example, the probability of violation in any individual day in order to have an expected number of 2 violations per year is 2/365 or .0055. From Figure 4 it can be shown that a .0055 probability of violation on an individual day produces a probability of exceeding more than two days per year of approximately .33. In order to reduce the probability of more than 2 violations in a year to approximately 1 percent, it would require a probability of violation on any individual day of .0016, again from Figure 4. At a probability of .0016, the expected number of violations in a year, over a long-term average, is .0016 x 365 or 0.58. Obviously, in order to generate a very low probability of exceeding a given number of days within a particular year, the average number of violations must be reduced far below that number.

The second statistical characteristic is that a binomial distribution is very sensitive to the number of possible violations within the time frame of the regulatory standard. Consider for example the effect of using an hourly standard as opposed to a daily one. For an hourly measurement, the standard is 365 x 24 or 8760 possible chances for violation within a year. Suppose the standard were written in a way that the probability of exceeding 2 or more days within a year had to be less than 10 percent. Under these conditions Figure 4 shows the expected probability of violation on any day would have to be .003. If hourly measurements were taken, then the probability of violation within any given hour would be .000043. Thus, the probability of violation within any given time frame is extremely sensitive to the number of possible violations.

2.3 Conversion of Coal Sulfur to Sulfur Dioxide

As previously stated, the principal objective of this report is to provide data and information to assist EPA in its analysis of the impact of coal sulfur variability on compliance with sulfur dioxide emissions-limiting regulations. However, translating coal quality analyses to potential emissions presents several problems.

First, as discussed in this section of the report, sulfur dioxide emission-limiting regulations vary with respect to the units of measure (lbs SO_2/MMBtu, ppm SO_2, percent sulfur, etc.) in which the limitations are expressed. As such, comparisons with a specific regulation require conversion to the appropriate measurement units.

Second, translating coal analysis data to potential or theoretical emissions requires some assumptions concerning the combustion characteristics of the coal. One of the most important assumptions concerns the conversion of coal sulfur to flue gas sulfur dioxide (SO_2), which may subsequently react with more oxygen, forming sulfur trioxide (SO_3) or sulfate radicals in a complex equilibrium. These reactions, combined with the fact that coal sulfur exists in various forms (pyritic, sulfate, and organic), result in only a portion of the total sulfur in coal being emitted as SO_2.

Various studies reviewed by EPA have indicated that for bituminous and subbituminous coals approximately two percent of the coals' sulfur is retained in the fly ash, about two percent is converted to SO_3, and about one percent is retained in the slag or bottom ash. Thus, as an approximation, about 95 percent of the total sulfur in coal is emitted as SO_2 from an uncontrolled boiler. It should be noted, however, that in specific cases, conversion factors of less than 90 percent were observed.1/

In the case of lignite coals, the various studies reviewed by EPA have indicated conversion factors ranging from 98 percent to as low as 50 percent. The wide range of conversion rates is related to the presence of reactive alkali substances (sodium, calcium, magnesium, and potassium) and/or clay and silica contents. In general, the alkali substances decrease the conversion rate while increased clay and silica contents increase the conversion rates.

The above discussion indicates that any assumption which attempts to generalize the conversion of coal sulfur to SO_2 emissions may be tenuous. This problem of estimating exhaust stack sulfur dioxide emissions from coal characteristics is further compounded by the incidental effects of coal processing equipment and control equipment not specifically designed for sulfur removal.

The first situation occurs when a plant utilizes a type of coal pulverizer which rejects a portion of the pyritic sulfur contained in the coal. Although the sulfur removal efficiency of such pulverizers is low compared to conventional coal cleaning facilities, in some cases, involving

---

1/ USEPA, Office of Air Quality Planning and Standards, Emission Standards and Engineering Division, "Background Information: Fuel Analysis Provisions for Performance Testing and Emission Monitoring of Sulfur Dioxide Emissions from Fossil Fuel Fired-Steam Generators", January 1977, Draft.

coals with high pyritic sulfur contents, the results may be significant.

The second situation relates to control equipment designed for removal of particulates. Although not specifically designed to control sulfur dioxide emissions, particulate control equipment frequently removes significant portions of the sulfur retained in the fly ash.

In view of these various problems relating to the estimation of potential sulfur dioxide emissions, the calculation of emissions in this report is based on the ratio of sulfur to heat content (lbs S/MMBtu) indicated by the laboratory analyses of coal samples.

2.4 Effect of the Choice of Statistical Distribution on Compliance with Sulfur Dioxide Emissions Regulations

In this report frequency distributions were analyzed to determine which type of distribution -- normal, lognormal, or inverted gamma -- most closely fit the observed distributions. Some of the previous studies on coal sulfur variability assumed a normal distribution, primarily due to the lack of adequate data and the statistical simplicity of the normal distribution. This section addresses the consequences of the choice of a particular distribution, as they relate to compliance with emissions regulations.

The methodology of this analysis included the calculation of the mean or average lbs SO_2/MMBtu required to meet sulfur dioxide emissions standards ranging from 1.0 to 6.0 lbs SO_2/MMBtu, based on relative standard deviations (RSD's) of the lbs SO_2/MMBtu ranging from 5 to 30 percent.[1] In addition, it was assumed that the stringency of the regulations required only a 0.5 percent probability of exceeding the standard on a given day. These calculations are set out in Tables 10, 11, and 12 for the respective normal, lognormal, and inverted gamma distributions.[2]

Table 10, for example, shows that under the assumption of a normal distribution an emissions regulation of 1.2 lbs SO_2/MMBtu would require a mean emissions of 1.063 lbs SO_2/MMBtu or less, if the emissions had an RSD of 5 percent. As the RSD of the emissions increases to 30 percent, and beyond, successively lower mean lbs SO_2/MMBtu are required for compliance.

---

[1] The relative standard deviation (RSD) is the ratio of the standard deviation to the mean, expressed as a percentage. The RSD provides a measure of the relative dispersion about the mean and is also called the coefficient of variation or coefficient of dispersion.

[2] The derivation of the equations used for the development of Tables 10, 11, and 12 is set out in Appendix A.

TABLE 10

NORMAL DISTRIBUTION

Calculation of the mean lbs SO_2/MMBtu required to meet a given standard, assuming a normal distribution, a 0.5% probability of exceeding the standard on an individual day, and a given relative standard deviation for the averaging period.

Emissions Standard (Lbs SO_2/MMBtu)	Relative Standard Deviation of Lbs SO_2/MMBtu (%)					
	5	10	15	20	25	30
	(Mean Lbs SO_2/MMBtu Required to Meet Emissions Standards)					
1.0	.886	.795	.721	.660	.608	.564
1.2	1.063	.954	.866	.792	.730	.677
1.4	1.240	1.113	1.010	.924	.852	.790
1.6	1.417	1.272	1.154	1.056	.973	.903
1.8	1.595	1.431	1.298	1.188	1.095	1.015
2.0	1.772	1.590	1.443	1.320	1.217	1.128
2.5	2.215	1.988	1.803	1.650	1.521	1.410
3.0	2.658	2.386	2.164	1.980	1.825	1.692
3.5	3.101	2.783	2.525	2.310	2.129	1.974
4.0	3.544	3.181	2.885	2.640	2.433	2.256
4.5	3.987	3.578	3.246	2.970	2.737	2.538
5.0	4.430	3.976	3.607	3.300	3.041	2.820
5.5	4.872	4.373	3.967	3.630	3.346	3.103
6.0	5.315	4.771	4.328	3.960	3.650	3.385

Source: Foster Associates, Inc.

TABLE 11

LOGNORMAL DISTRIBUTION

Calculation of the mean lbs SO_2/MMBtu required to meet a given standard, assuming a lognormal distribution, a 0.5% probability of exceeding the standard on an individual day, and a given relative standard deviation for the averaging period.

Emissions Standard (Lbs SO_2/MMBtu)	Relative Standard Deviation of Lbs SO_2/MMBtu (%)					
	5	10	15	20	25	30
	(Mean Lbs SO_2/MMBtu Required to Meet Emissions Standard)					
1.0	.880	.777	.689	.612	.547	.490
1.2	1.056	.933	.826	.735	.656	.588
1.4	1.232	1.088	.964	.857	.765	.686
1.6	1.409	1.244	1.102	.980	.875	.784
1.8	1.585	1.399	1.239	1.102	.984	.882
2.0	1.761	1.555	1.377	1.225	1.093	.980
2.5	2.201	1.943	1.721	1.531	1.367	1.225
3.0	2.641	2.332	2.066	1.837	1.640	1.470
3.5	3.081	2.720	2.410	2.143	1.913	1.715
4.0	3.521	3.109	2.754	2.449	2.187	1.961
4.5	3.961	3.498	3.099	2.755	2.460	2.206
5.0	4.402	3.886	3.443	3.062	2.733	2.451
5.5	4.842	4.275	3.787	3.368	3.007	2.696
6.0	5.282	4.664	4.132	3.674	3.280	2.941

Source: Foster Associates, Inc.

TABLE 12

INVERTED GAMMA DISTRIBUTION

Calculation of the mean lbs SO_2/MMBtu required to meet a given standard, assuming an inverted gamma distribution, a 0.5% probability of exceeding the standard on an individual day, and a given relative standard deviation for the averaging period.

Emissions Standard (Lbs SO_2/MMBtu)	Relative Standard Deviation of Lbs SO_2/MMBtu (%)					
	5	10	15	20	25	30
	(Mean Lbs SO_2/MMBtu Required to Meet Emissions Standard)					
1.0	.877	.767	.678	.596	.528	.461
1.2	1.053	.920	.813	.716	.633	.553
1.4	1.228	1.073	.949	.835	.739	.645
1.6	1.404	1.227	1.084	.954	.844	.737
1.8	1.579	1.380	1.220	1.073	.950	.829
2.0	1.755	1.533	1.356	1.193	1.055	.921
2.5	2.193	1.916	1.694	1.491	1.319	1.152
3.0	2.632	2.300	2.033	1.789	1.583	1.382
3.5	3.070	2.683	2.372	2.087	1.847	1.613
4.0	3.509	3.066	2.711	2.385	2.111	1.843
4.5	3.948	3.450	3.050	2.683	2.375	2.073
5.0	4.386	3.833	3.389	2.981	2.639	2.304
5.5	4.825	4.216	3.728	3.279	2.902	2.534
6.0	5.264	4.600	4.067	3.578	3.166	2.764

Source: Foster Associates, Inc.

The impact of the choice of distributions can be assessed by a comparison of Tables 10, 11 and 12. For example, assume a coal determined to have an RSD of 5 percent and an emissions standard of 1.0 lbs SO_2/MMBtu. If the emissions were normally distributed, a mean of 0.886 lbs SO_2/MMBtu would be required for compliance. Alternatively, lognormal and inverted gamma distributions would require means of 0.880 and 0.877, respectively.

Additional comparisons of Tables 10, 11, and 12 show that the impact of the choice of distribution is almost insignificant when stringent emissions standards and coals with low variability are considered. However, as the stringency of the emissions standard declines and more importantly, as the variability increases, the difference becomes more significant. For example, based on a 5.0 lbs SO_2/MMBtu emission standard and a coal with an RSD of 30 percent the normal distribution would require a mean of 2.82 lbs SO_2/MMBtu for compliance, while the inverted gamma distribution indicates 2.30 lbs SO_2/MMBtu.

In summary, when highly variable coals and less restrictive emissions standards are considered, the choice of the frequency distribution of lbs SO_2/MMBtu becomes increasingly important. This analysis indicates that when dealing with the NSPS the choice among these three distributions is almost insignificant, since the differences among the means required for compliance in most cases are within the acceptable limits for error in the ASTM coal sampling and analysis procedures. However, in the case of SIPs which frequently permit a higher level of emissions, the choice of distribution cannot be ignored when a highly variable source of coal is utilized.

2.5 Theoretical Effects of Measurement Error on the Relative Standard Deviation of Pounds of Sulfur per Million Btu

As discussed in Section 2.3, sulfur emissions in this study were calculated as the ratio of sulfur content to heat content (lbs S/MMBtu), due to problems in translating coal sulfur contents to sulfur dioxide emissions. The calculation of lbs S/MMBtu requires separate measurements for the sulfur and heat contents of coal. Each of these measurements is subject to two sources of error. The first source of error is sampling error which may result in coal samples that are not representative of the true coal population. The second source of error arises from the analytical or laboratory techniques used to chemically analyze the coal samples.

This analysis is not based on observed data but instead examines the theoretical effects of measurement error in coal sampling and analysis resulting from ASTM standards and procedures. The analysis of the impact of measurement error was performed in a three-step process. First, estimates of the measurement and sampling error were developed for both sulfur and heat contents. Second, the mathematics required to determine how these errors affect the lbs S/MMBtu were derived. Finally, the results of the first two steps were used to calculate the impact of measurement error on coal sulfur variability, defined as the RSD of lbs S/MMBtu.

In the next two sections, measurement and sampling errors are derived and their effects upon the RSD of lbs S/MMBtu are examined. The derivation of the mathematical formula which describes the impact of measurement and sampling errors upon sulfur variability is set out in Appendix B.

2.5.1 Estimates of Sampling and Analytical Error

With respect to the analytical error for percent sulfur, the ASTM standards state:

"16.2 Reproducibility -- The means of results of duplicate determinations carried out by different laboratories on representative samples taken from the same bulk sample after the last stage of reduction should not differ by more than the following:

Coal containing less than 2% sulfur	0.10%
Coal containing 2% sulfur or more	0.20%[1/]"

RSD, as previously defined, is the standard deviation divided by the mean. Under the assumption that the error in percent sulfur is normally distributed and the standard deviation estimates represent approximately the 99th percentile (3 standard deviations), the RSD of the error introduced by analytical techniques for a coal with a sulfur content of 1.5 percent is calculated as:

$$\frac{0.1}{1.5 \times 3} = .022 \text{ or } 2.2 \text{ percent}$$

Applying these assumptions to various coal sulfur contents yields the following estimates of the RSD of the error introduced by analytical techniques:

[1/] ASTM D 3177-75, "Standard Test Methods for Total Sulfur in the Analysis Sample of Coal and Coke", 1978 Annual Book of ASTM Standards, Part 26, p. 399.

Coal Sulfur Content (%)	RSD of the Error Introduced by Analytical Techniques (%)
0.5	6.7
1.5	2.2
2.5	2.7
3.5	1.9
4.5	1.5

With respect to the measurement error in coal heat contents, the relevant ASTM standard says:

"13.1.2 Reproducibility -- The results submitted by two or more laboratories (different equipment, operators, date of test and different portions of the same pulp) should not be considered suspect unless the two results differ by more than 100 Btu/lb, dry basis."[1]

Based on a coal with an average heat content of 13,000 Btu per lb, a difference of 0.77 percent would not be suspect. Assuming a normal distribution and three standard deviations, the acceptable analytical error for heat content translates to an RSD of approximately 0.26 percent.

With respect to sampling error, the general purpose sampling procedure is intended to provide a precision of plus or minus one-tenth of the ash content of the coal sampled in 95 out of 100 cases.[2] Based on the general purpose sampling procedures, the precision intended for ash is also applicable to sulfur and heat contents. Thus, the RSD of

---

1/ ANSI/ASTM D 2015-77, "Standard Test Method for Gross Calorific Value of Solid Fuel by the Adiabatic Bomb Calorimeter", 1978 Annual Book of ASTM Standards, Part 26, p. 307.

2/ ASTM D 2234-76, "Standard Methods for Collection of a Gross Sample of Coal", 1978 Annual Book of ASTM Standards, Part 26, p. 310.

the sampling error for both sulfur and heat contents would result in a value of 5 percent.[1/]

As this discussion shows, the major source of variance is in the collection of gross samples. Note that these estimates assume the ASTM measurements are followed exactly. If they are not, the RSD estimates are likely to be larger.

2.5.2 Analysis of the Impact of Measurement Error on the RSD of Lbs S/MMbtu

The previous discussion shows that an evaluation of the impact of measurement error on the RSD of lbs S/MMBtu requires an assessment of the impact of four separate error terms:

- RSD of error in sulfur measurement due to sampling
- RSD of error in sulfur measurement due to analysis
- RSD of error in heat content measurement due to sampling
- RSD of error in heat content measurement due to analysis

The mathematical formula which describes the relationship of these error terms to measured and true RSD values was derived and was used to construct Table 13.

Table 13 sets out the estimates of the RSD of lbs S/MMBtu which would be calculated from coal sample analyses along with the true RSD after allowing for measurement error. Table 13 shows, for example, that if a coal with an average sulfur content of 2.5 percent exhibited an RSD of 10 percent based on ASTM samples and analyses, the true RSD is only 6.6 percent. The difference between the measured and true

---

[1/] Based on the most precise ASTM sampling classification which requires unbiased, stopped belt cross-section increments, spaced evenly in time.

RSD's (10.0-6.6 = 3.4) is attributed to measurement error. The relationship between the measured RSD and the individual error terms is such that the measured RSD is biased, resulting in overestimates of the true RSD. Note that there is little difference between high- and low-sulfur coals. Also, the difference between the measured and true RSD gets progressively larger as the measured RSD gets smaller. Calculations indicate that at a measured RSD of 7 to 8 percent, the true RSD approaches zero.

TABLE 13

COMPARISON OF MEASURED RSD WITH TRUE RSD
(Lbs S/MMBtu)

Measured RSD (%)	True RSD (%) by Level of Coal Sulfur Content (%) 0.5	1.5	2.5	3.5	4.5
8	--	3.0	2.7	3.2	3.4
10	2.5	6.7	6.6	6.8	6.9
12	7.1	9.4	9.3	9.5	9.6
14	10.1	11.8	11.7	11.9	12.0
16	12.7	14.2	14.1	14.2	14.3
18	15.1	16.4	16.3	16.4	16.5
20	17.5	18.5	18.5	18.6	18.7
22	19.7	20.7	20.6	20.7	20.8
24	21.9	22.8	22.7	22.8	22.9
26	24.1	24.9	24.8	24.9	25.0
28	26.3	27.0	26.9	27.0	27.0
30	28.4	29.0	29.0	29.1	29.1
32	30.5	31.1	31.1	31.1	31.2
34	32.6	33.1	33.1	33.2	33.2
36	34.7	35.2	35.2	35.2	35.3
38	36.7	37.2	37.2	37.2	37.3
40	38.8	39.3	39.2	39.3	39.3
42	40.9	41.3	41.3	41.3	41.4
44	42.9	43.4	43.3	43.3	43.4
46	45.0	45.4	45.4	45.4	45.4

Source: Foster Associates, Inc.

3.0 Methodology

3.1 Data Collection

The approach used in this study consisted of two phases, data collection and data analysis. In the first phase, all relevant data pertaining to coal sulfur variability were collected and consolidated in a computer data base. The majority of the data sets in the data base reflect data acquired with the assistance of the Edison Electric Institute and have not been previously analyzed for coal sulfur variability.

Electric utilities, coal companies, and the Bureau of Mines were the primary sources of the data included in this study. Separate meetings were held with representatives of the electric utility and coal industries to solicit data and to provide comments and recommendations for the proposed study. Based on these discussions, potential sources of data were identified and contacted.

In the data collection phase of the study, electric utilities, coal companies, and other organizations1/ were contacted. As set out below, 26 electric utilities, five coal companies, and five other organizations responded with data.

Type of Companies (1)	Number of Companies Contacted (2)	Responded (3)
Electric Utilities	69	26
Coal Companies	17	5
Other 1/	8	5
Total	94	36

1/ Other organizations include research organizations, Federal agencies, and industrial companies.

Data collected in this study include stack monitoring data and coal analysis data. Stack monitoring data relate to the sulfur dioxide emissions (ppm or lbs SO_2/MMBtu) present in the exhaust gases resulting from combustion.

Coal analysis data reflect the measured physical and chemical coal characteristics, determined from coal samples sent to analytical laboratories.

A problem frequently encountered in the collection of data, especially for coal analyses, was that companies did not maintain data of the quality desired for this study. In order to isolate those factors which may contribute to coal variability, the following information was requested for each data set:

A. Per coal source of supply represented by analyses
  1. Coal source
    a. Bureau of Mines Producing District
    b. State
    c. County
    d. Seam(s)
    e. Mine
  2. Method of sampling (automatic or hand, ASTM or non-ASTM)
  3. Type of sample (core, as mined, as delivered, as burned)
  4. Analysis method (ASTM or other)
  5. Degree of processing (run-of-mine, washed, stoker, etc.)
  6. Mining method (surface or underground)

B. Per coal cample analysis
   1. Date of sample or date of coal delivery
   2. Lot-size (tons)
   3. Sulfur content (percent)
   4. Heat content (Btu/lb)
   5. Ash content (percent)
   6. Moisture content (percent)

Additionally, it was requested that each coal analysis data set be restricted to coal analyses from one mine or one coal seam and that ASTM sampling and analysis procedures were preferred.

Particularly in the Appalachian producing areas, it was found that the availability of data which satisfied these criteria was extremely limited. In general, coal production in this area is based on relatively small, multiple seam mines and the coal is frequently shipped by truck directly to the consumer, or is shipped to a central loading point for volume or unit train shipments. In the case of truck shipments, the coal is generally only spot checked by hand sampling, and in the case of volume or unit train shipments, the analyses represent a mixture of various unidentified seams and mines.

One area of special interest in this study was an examination of coal sulfur variability with respect to lot-size. Considerable effort was directed toward the acquisition of smaller lot-size samples, such as for individual railroad cars (about 80 to 100 tons) or for the composite of several railroad cars. In practice, however, it was found that few companies perform routine sampling and analysis according to ASTM procedures for such small lot-sizes. The smallest lot-sizes for which routine ASTM sampling and analysis were found available from electric utilities and coal companies were approximately 750 or 1,500 tons, which correspond to the capacities of regular- and jumbo-size barges.

The majority of the data sets based on ASTM sampling and analysis procedures reflect unit-train size shipments (approximately 10,000 tons) from the Mid-Continent and Western producing areas. These data sets preclude an analysis of small lot-size or short-term coal sulfur variabilities since a typical plant of 500 MW capacity will burn about 200 tons per hour or approximately one unit-train over a period of two days.

Coal analysis data obtained from the Bureau of Mines provided data necessary to address the problem of short-term coal sulfur variability. These data were obtained from the detail records of the Bureau of Mines "Current Coal History" data tape, which contained analyses for the period 1966 to 1978. Coal analyses on this data tape reflect coals purchased by Federal installations and the gross samples of these coals represent volumes of 1,000 tons or less, as specified by the Bureau of Mines sampling procedures. It should be noted that these coals are not necessarily comparable to the coals consumed by electric utilities. A significant portion of the Bureau of Mines data are based on washed, double-screened coals which are more representative of coals consumed by industrial plants.

No attempt was made to obtain random samples of coals from different producing regions, seams, or mines. However, in the data collection phase of the project, special emphasis was directed toward obtaining representative data sets from each of the Bureau of Mines Producing Districts.

3.2 Data Base

The data base developed in this study consists of approximately 116,000 records, each of which represents an individual coal analysis or stack monitoring observation. The data base is divided into two sub-categories -- the coal analysis file and the stack monitoring file.

3.2.1 Coal Analysis Data File

The coal analysis portion of the data base consists of approximately 94,700 records and represents coal samples from more than 1,200 mines or combinations of mines. The ranks of coals included in the data base are lignite, subbituminous, and bituminous. The analyses represent production samples and core analyses collected in recent years, primarily between 1966 and 1978. The coals from which these samples were obtained were primarily steam coals delivered to electric utility steam generating plants and Federal installations. A relatively minor portion of this data base contains coal analyses from shipments to industrial plants.

Appendix C sets out the data base format for the coal sulfur analysis data. Included in Appendix C are the field descriptions of the data base and a brief description of the relevant parameters used in classifying the data.

3.2.2 Stack Monitoring Data File

The second portion of the data base consists of the stack monitoring data file. This portion consists of approximately 21,500 records for continuous monitoring data from six electric generating units and one industrial unit. The general format for the stack monitoring data is set out in

Appendix D. It should be noted that the available data sets did not permit the documentation of all the factors set out in the stack monitoring format. For example, some of the data sets had no record of the gross load (MWH) or the coal flow (tons/hr) to relate to the emissions data.

3.2.3 Index to Mine Locations and Seams Produced

Set out in Appendix E is an index to mines and seams included in the data base developed in this study.1/ Each mine or source of coal has been assigned a five-digit mine code number as set out in Column (1) of Appendix E. When the analyses were reported as a composite of multiple mines, the composite source was assigned a mine code and the individual mines included in the composite were set out in Column (2), "Blend of Mines." Column (2) generally identifies the multiple sources of coal in a stockpile which relate to stack monitoring data or coal analyses based on "as burned" or "as fired" samples.

Columns (3) and (4) identify the respective state and county locations of the various mines for which analyses were obtained.

Columns (5) through (7) in Appendix E identify the seams associated with each of the individual mines or sources. The "Reference Code" in Column (5) is the seam code number identified in the computer data base. These six-digit codes are interpreted as follows:

1/ Appendix E is applicable only to data collected by Foster Associates. Bureau of Mines data were not tabulated due to the large number of mines (approximately 1,000), many of which contain only a few coal analyses.

First digit: A zero indicates that there exists an equivalent Bureau of Mines' code for the seam or combination of seams identified. Codes beginning with the numeral "1" indicate that a Bureau of Mines' code for the seam(s) does not exist.

Digits 2-4: In the case of codes beginning with a zero, the second through fourth digits reflect the three-digit Bureau of Mines' codes assigned to specific seams or combinations of seams.

Digits 5-6: In the case of codes beginning with a zero, the fifth and sixth digits are identifiers for various local names assigned to the same coal seam such as No. 8, Pittsburgh, Big Vein, etc.

Digits 2-6: For codes beginning with the numeral "1", the second through sixth digits represent unique codes assigned to the seams or combination of seams for which no Bureau of Mines' codes exist.

Column (6) identifies the name of the seam or seams represented by the code in Column (5). When no Bureau of Mines' code existed for a combination of seams, the Bureau of Mines' codes for each of the individual seams were provided in Column (7) as supplemental information.

3.2.4 Maps of Mine Locations by Producing District, State, and County

The geographical location of each of the mines or sources of coal is set out in Appendix F. The maps identify the Bureau of Mines Producing District by State and County. For each of the Producing Districts, the individual mines and sources of coal are identified by county of origin. For each county, the total number of mines as well as the codes of individual mines are identified.

The relevant seam information for each of the mines located on the maps may be obtained by cross-reference to Appendix E.

3.3 Analysis of Data

The second phase of this study analyzed the data received from coal companies, electric utilities, Bureau of Mines and other sources. The objectives of this study and the magnitude of the data bases dictated the use of computerized statistical programs. The output of these programs was not a final product but rather provided the necessary statistical data, in a summarized format, for further comparisons and analyses.

The analytical program, which consists of various subprograms, is discussed in the next section. Although the discussion relates to coal analysis data, minor modifications to the program permitted a similar analysis of stack monitoring data.

Set out in Appendix G is an example of the computer output of the programs developed for the analysis of coal data. A separate analysis was performed for each of the following variables in the data sets analyzed: volume, sulfur content, heat content, and pounds of sulfur per MMBtu (lbs S/MMBtu).

The volume variable represents the size (lot-size) of the shipment from which the coal samples and analyses were obtained and is expressed in tons. Sulfur contents and heat contents were analyzed on an "as received" basis, with the exception of several data sets for which only "dry" basis analyses were available. The variable lbs S/MMBtu was calculated based on the sulfur and heat contents reported in the coal analysis data. As previously indicated, no attempt was made to estimate potential exhaust stack emissions resulting from coal combustion due to the inherent difficulties and variables associated with plant-specific coals and equipment.

Each of the four variables (volume, sulfur content, heat content, and lbs S/MMBtu) was analyzed in a computer routine which: (1) plotted the variable as a function of time and provided the mean, standard deviation, and relative standard deviation (RSD); (2) plotted frequency distributions for the observed data; (3) compared the observed frequency distributions to the expected frequency distributions based on normal, inverted gamma, and lognormal distributions, and (4) compared the goodness of fit between the observed and the expected distributions. Each of these analyses is explained below and is illustrated in Appendix G.1/

3.3.1 Plot of Variable vs. Time

A time plot of each of the variables was constructed to visually examine possible variations during the chronological sequence of the data. With respect to volumes, this routine chronologically plots the volume of coal represented by each analysis and permits a visual examination for consistency in lot-sizes. For sulfur content, heat content, and lbs S/MMBtu, this plot permits a visual examination for possible

---

1/ The analyses performed in this study were based on a simple model assuming independent variance. Budget constraints did not permit an investigation of the merits of more sophisticated autocorrelative models.

autocorrelation in the data or indications of some external factor, such as a change in mining method or coal preparation, which has produced shifts in the data population.

3.3.2 Sample Statistics

The mean, standard deviation, and relative standard deviation were calculated for each of the variables analyzed. Included in the statistical program was a routine which flagged any analysis which had a value which exceeded plus or minue five standard deviations from the mean. Each of these flagged records was manually compared to the raw data to check for possible keypunch or transcription errors.

3.3.3 Frequency Distribution of Observed Data

In the frequency distribution routine of the analytical program, nine equal intervals or cells were defined, based on the sample statistics (mean and standard deviation) of the data set. A histogram of each frequency distribution was constructed and the number of observations contained within each of the nine cells was recorded.

3.3.4 Comparison of Observed Distribution to Expected Distribution

The next step in the analytical program was to compare the observed frequency distributions to the expected frequency distributions of the normal, inverted gamma, and lognormal distributions. The selection of these three distributions for comparative purposes, was based upon findings of previous studies on coal sulfur variability and discussions with personnel from EPA and the electric utility and coal industries. Although there appeared to be no consensus of opinion on the distribution of sulfur in coal or the distribution of lbs S/MMBtu, these three distributions were the ones most frequently encountered in discussions of coal sulfur variability and represent a reasonable range of alternative distributions.

Set out in Table 14 is a sample comparison for a data set consisting of 1,537 analyses for a Wyoming subbituminous coal. Column (1) sets out the cell number for the nine cells generated in the histogram, while the limits of each of these cells are identified in Column (2). Column (3) summarizes the number of observations occurring in each of the cells and Columns (4), (5), and (6) reflect the expected number of observations within each cell based upon normal, inverted gamma, and lognormal distributions, respectively. Figure 6 graphically displays the relationship between the

TABLE 14

COMPARISON OF OBSERVED AND EXPECTED FREQUENCY DISTRIBUTIONS OF LBS S/MMBTU FOR A WYOMING SUBBITUMINOUS COAL (CORE ANALYSES)

		Number of Observations (N = 1,537)			
			Expected Distributions		
Cell Number (1)	Cell Limits (Lbs S/MMBtu) (2)	Observed (3)	Normal (4)	Inverted Gamma (5)	Log-normal (6)
1	0.00-0.20	0	0.4	0.0	0.0
2	0.20-0.28	0	9.2	0.3	20.0
3	0.28-0.36	57	93.1	62.8	54.6
4	0.36-0.44	502	371.5	450.5	417.3
5	0.44-0.53	498	588.6	598.1	620.2
6	0.53-0.61	387	371.5	306.1	310.0
7	0.61-0.69	77	93.1	92.7	95.3
8	0.69-0.77	12	9.2	21.2	16.7
9	0.77	4	0.4	4.2	2.9

observed data and the expected values based on the normal, inverted gamma, and lognormal distributions.

As shown on Figure 6, visually there appears to be no significant differences among the three expected distributions. However, given the requirement of a very low probability of exceeding an emissions level combined with highly variable coal sulfur contents, the type of distribution can have a significant impact on compliance as discussed in previous sections of this report.

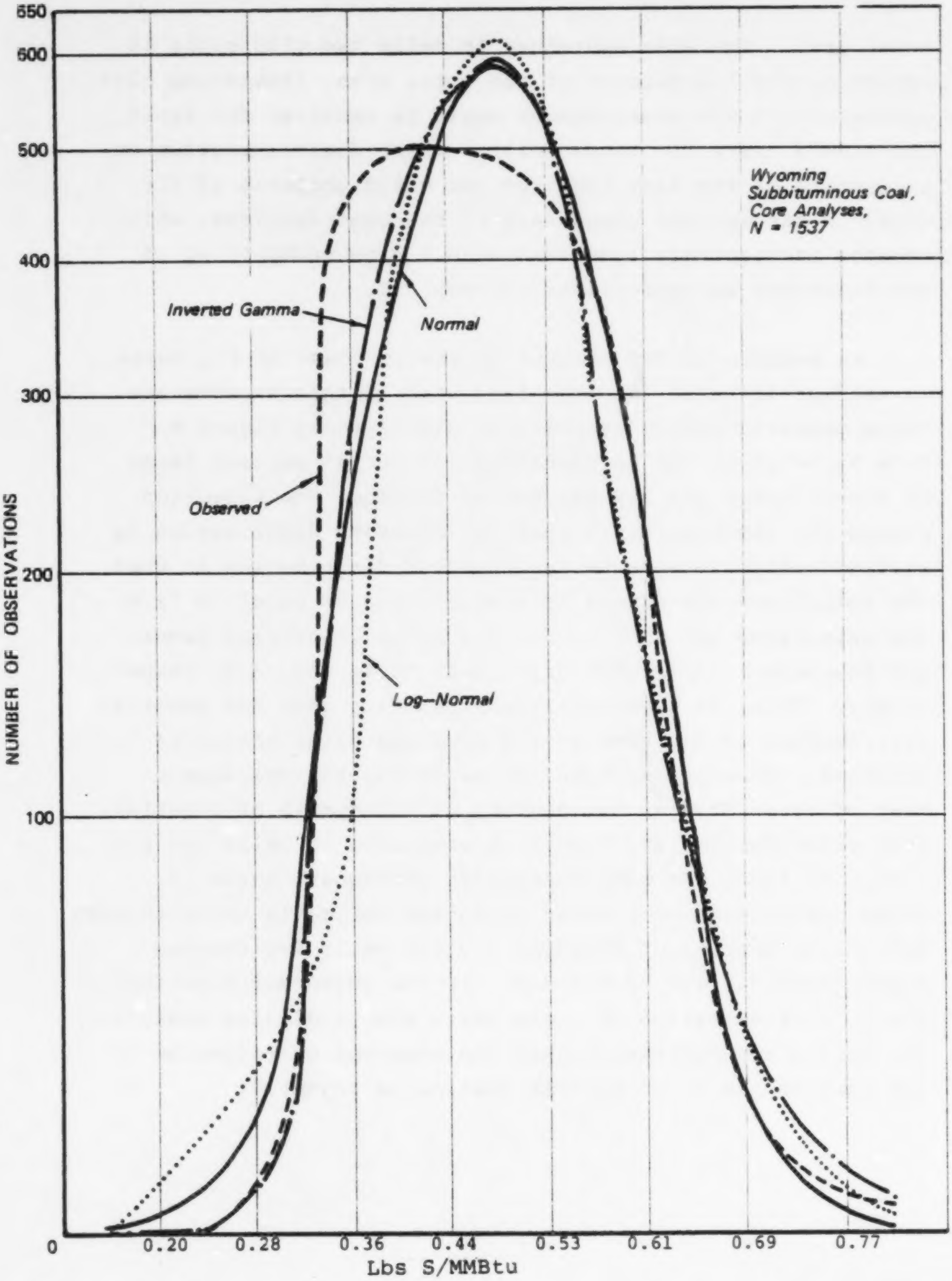
3.3.5 Goodness of Fit Between the Observed and Expected Frequency Distributions

The final routine in the analytical program addressed the problem of goodness of fit between the observed distribution and the expected values based on the normal, inverted gamma, and lognormal distributions. The chi-square test of statistical significance was used to determine which, if any, of the expected distributions approximated the distribution of the observed data.

The chi-square test was used to analyze the goodness of fit for cells one through nine of the histogram (six degrees of freedom), cells two through eight (four degrees of freedom), and cells three through seven (two degrees of freedom). These last two tests were used to exclude the end cells of the distributions, which frequently had less than the five observations per cell necessary for a valid chi-square test. Under the assumptions of a normal distribution, approximately 20,000 observations would be required to obtain an expected value of 5 observations each in cells one and nine. The area under the normal distribution contained in cells one and nine is equal to approximately 0.05 percent of the

FIGURE 6

RELATIONSHIP BETWEEN OBSERVED DATA AND EXPECTED VALUES BASED ON NORMAL, INVERTED GAMMA, AND LOGNORMAL DISTRIBUTIONS



total area. The area contained in cells two plus eight is approximately 1.2 percent of the total area, indicating that approximately 800 observations would be required for valid chi-square tests for these cells. These figures provide an indication of the data required for valid goodness of fit tests in the extreme right-tail of the distributions, which becomes increasingly important when a high probability of not exceeding an upper limit is required.

An example of the results of the goodness of fit tests is set out in Table 15. The data used in this example are those observed values graphically displayed in Figure 6. From Table 15 it can be seen that, at the 95 percent level of significance and six degrees of freedom, the rejection region for the hypothesis that the observed distribution is statistically the same as the expected distribution is when the calculated chi-square is greater than or equal to 12.6. The calculated chi-squares for the normal, inverted gamma, and lognormal comparisons are 113.4, 50.4, and 72.6, respectively. Thus, in each case the hypothesis that the observed distribution is the same as the expected distribution is rejected. However, as noted on Table 15, the chi-square test of these data at six degrees of freedom is of questionable value due to less than 5 observations in cells one and nine. In fact, the only conclusive chi-square tests in these comparisons were those performed for cells three through seven (two degrees of freedom) and for cells two through eight (four degrees of freedom) for the lognormal distribution. An examination of these tests also indicates that, in all cases, the hypothesis that the observed distribution is the same as the expected distribution is rejected.

TABLE 15

COMPARISON OF GOODNESS OF FIT OF OBSERVED DATA WITH THE EXPECTED NORMAL, INVERTED GAMMA AND LOGNORMAL DISTRIBUTIONS

Type of Distribution	Number of Observations in Cell Number: 1	2	3	4	5	6	7	8	9
Normal - Observed	0	0	57	502	498	387	77	12	4
Normal - Expected	0.4	9.2	93.1	371.5	588.6	371.5	93.1	9.2	0.4
Inverted Gamma-Observed	0	0	57	502	498	387	77	12	4
Inverted Gamma-Expected	0	0.3	62.8	450.5	598.1	306.1	92.7	21.2	4.2
Lognormal - Observed	0	9	70	448	514	419	67	6	4
Lognormal - Expected	0.4	9.2	93.1	371.5	588.6	371.5	93.1	9.2	0.4

Chi-Square Test

Hypothetical Distribution	Calculated Chi-Square* 6 Degrees of Freedom	4 Degrees of Freedom	2 Degrees of Freedom
Normal	113.4a/	84.9a/	76.1
Inverted Gamma	50.4a/	50.3a/	46.4
Lognormal	72.6a/	44.1	43.3

* Rejection Regions, 0.95 level of significance:
6 Degrees of Freedom, Chi-Square ≥ 12.6
4 Degrees of Freedom, Chi-Square ≥ 9.5
2 Degrees of Freedom, Chi-Square ≥ 6.0

a/ Chi-Square test is of questionable value with less than five observations in certain cells.

Source: Foster Associates, Inc.

Although all the calculated chi-square values in Table 15 indicate that the observed distributions are statistically different from the three distributions used for comparison, it is still possible to make a general statement about the goodness of fit.[1] Based on the chi-square values calculated at two degrees of freedom, the distribution of the observed data can be ranked according to goodness of fit as:

TABLE 16

RANKING OF GOODNESS OF FIT BY CHI-SQUARE TEST

Rank	Distribution	Chi-Square[1]
1.	lognormal	(43.3)
2.	inverted Gamma	(46.4)
3.	normal	(76.1)

[1] The lower the chi-square, the better the fit.

At two degrees of freedom, the two end cells for each tail of the distribution were not considered, thus the distributions were compared or ranked only for the central portion of the distributions accounting for approximately 99.15 percent of the total area under the curve in the case of a normal distribution.

---

[1] This general statement about the goodness of fit is based on the assumption that for each cell the square of the absolute difference between the observed and expected frequencies divided by the expected frequency is a valid measure of goodness of fit. In later sections of this report the goodness of fit analysis is used to examine the fit for extreme values, or the tails of the frequency distributions.

4.0 Analysis of Coal Data Received from Respondents

This section relates to the analysis of the coal data provided by respondents and specifically excludes data obtained from the Bureau of Mines. The analysis of Bureau of Mines coal data is discussed in Section 5.0 of this report.

In order to better understand the structure of this section of the report, a brief discussion of the topics contained herein and an overview of the methodology are helpful. Three general topics are included in this section: (1) sample statistics, (2) predictability of mine variability based on composite coal seam and Producing District data, and (3) choice of statistical distribution.

The section on sample statistics discusses the variabilities of lbs S/MMBtu exhibited by the coal data sets at various levels of aggregation. The methodology consisted of three general levels of analysis. First, the individual data sets received from the respondents were analyzed. These data sets generally reflect coal analyses from individual mines, and in many cases, mines producing from a single coal seam. With respect to sample statistics (mean, standard deviation, and RSD), this analysis summarized the entire data set with a single RSD of lbs S/MMBtu and encompassed a wide range of lot-sizes. Second, the individual data sets were examined according to lot-sizes. This involved the generation of sub-data sets, sorted by specific lot-size intervals. For example, a data set may have contained coal analyses reflecting shipments ranging from 1,000 to 10,000 tons. Sorting by lot-size provided new data sets representing smaller intervals such as 1,000 to 2,000 tons, 2,000 to 3,000 tons, etc. This permitted an assessment of the influence of lot-size on the variability (RSD of lbs S/MMBtu) within individual mines. Finally, a series of special aggregate analyses was performed. This included analyses of data by Producing District, by coal seam, by degree of preparation, and other factors which may contribute to variability.

The second topic, which examines the predictability of mine variability from coal seam and Producing District data, utilizes the results from the statistical analyses of individual mines, aggregate seams and Producing Districts. These results are analyzed and compared to determine to what extent coal sulfur variabilities can be predicted or generalized.

The third topic examines the choice of statistical distributions based on the results of the chi-square routine of the analytical program, comparisons of relative distribution error and comparative plots of the observed and expected data for the right-tail of the frequency distributions. This analysis examines various coals and discusses which frequency distributions best fit the observed data.

4.1 Sample Statistics

In general, each of the data sets gathered by Foster Associates, PEDCo, and EPA were analyzed through the use of the analytical program. Appendix H sets out the salient characteristics of these data sets which, as previously indicated, reflect coal analyses from individual mines and in many cases, mines producing a single coal seam. In certain cases, due to an inadequate number of observations in the data set, the individual data sets were not separately analyzed but used only for aggregate analyses for specific seams or producing districts.

A series of analyses was also performed on the data at various levels of aggregation to examine the impact of the variables which have been identified as possible factors contributing to coal sulfur variability.

4.1.1 Individual Data Sets

The first step in the analysis of the data gathered in this study focused on the data as supplied by the respondents. The results of this analysis by USBM Producing District are set out in Appendix I. Cross-reference of the mine codes in Appendix I to the information in Appendix H provides a detailed description of the documented parameters (rank, mining method, preparation, etc.).

In this analysis 205 separate data sets, representing 18 Producing Districts, were analyzed. Since these data sets reflect sequential deliveries from the same mine or source of supply, many of the factors which may influence coal sulfur variability have been isolated and may be assumed constant. These factors include geographical location, seam, rank and type of sample. Other factors, such as lot-size, mining method, degree of preparation, and method of sampling may vary within the individual data sets but in many cases may exhibit sufficient consistency to be assumed constant.

The coals in these data sets are almost entirely steam coals, produced for the electric utility market. In general, these coals are crushed run-of-mine coals but in some cases, such as in the Mid-Continent area (Illinois, Indiana, and western Kentucky), the coals are predominantly washed.

Appendix I illustrates the variations in coal characteristics actually experienced by electric utilities. Most of these data sets represent coal deliveries under term contracts. Columns (4) and (5), for example, illustrate the mean and the variation in the size of individual coal shipments delivered. In general, the RSD of the shipment size ranges from 20 to 60 percent. Many of the factors contributing to these variations, such as strikes, transportation availability, and bad weather, are beyond the control of the consumer. One exception to the large variations in lot-size occurs in Producing District 19 where uniform shipments of approximately 10,000 tons (unit train) are predominant. Here the RSD's of the lot-sizes are generally less than 10 percent.

A summary of the ranges of lbs S/MMBtu based on the means and RSD's is set out in Table 17. In this summary only 140 data sets, each containing 30 or more observations, were considered.[1] The average lbs S/MMBtu ranged from 0.19 for a source in Producing District 16 (Colorado) to 4.99 for a source in Producing District 4 (Ohio). The RSD's of the lbs S/MMBtu ranged from 2.1 percent for a source in Producing District 10 (Illinois) to 67.2 percent for a source in Producing District 4 (Ohio).

4.1.1.1 RSD Versus Lot-Size

The results of this first analysis of data were examined for indications of any relationship between the RSD of the lbs S/MMBtu and the average lot-size of the data sets.[2]

---

[1] RSD and other statistics computed with less than 30 observations are subject to a large sampling error and may not be representative of the true population.

[2] In this analysis, the average lot-size of each data set was plotted against the RSD of the lbs S/MMBtu. In later analyses, data sets exhibiting large variations in lot-sizes were aggregated by specific lot-size intervals within the data set.

The expected inverse relationship between RSD and lot-size was not exhibited in this analysis. An example of the results of this analysis is set out in Figure 7, which provides a comparison of the RSD's and average lot-sizes for data sets from the Mid-Continent producing area (western Kentucky, Illinois, and Indiana).

TABLE 17

SUMMARY OF THE AVERAGE LBS S/MMBTU
AND RSD FOR INDIVIDUAL DATA SETS BY PRODUCING DISTRICT

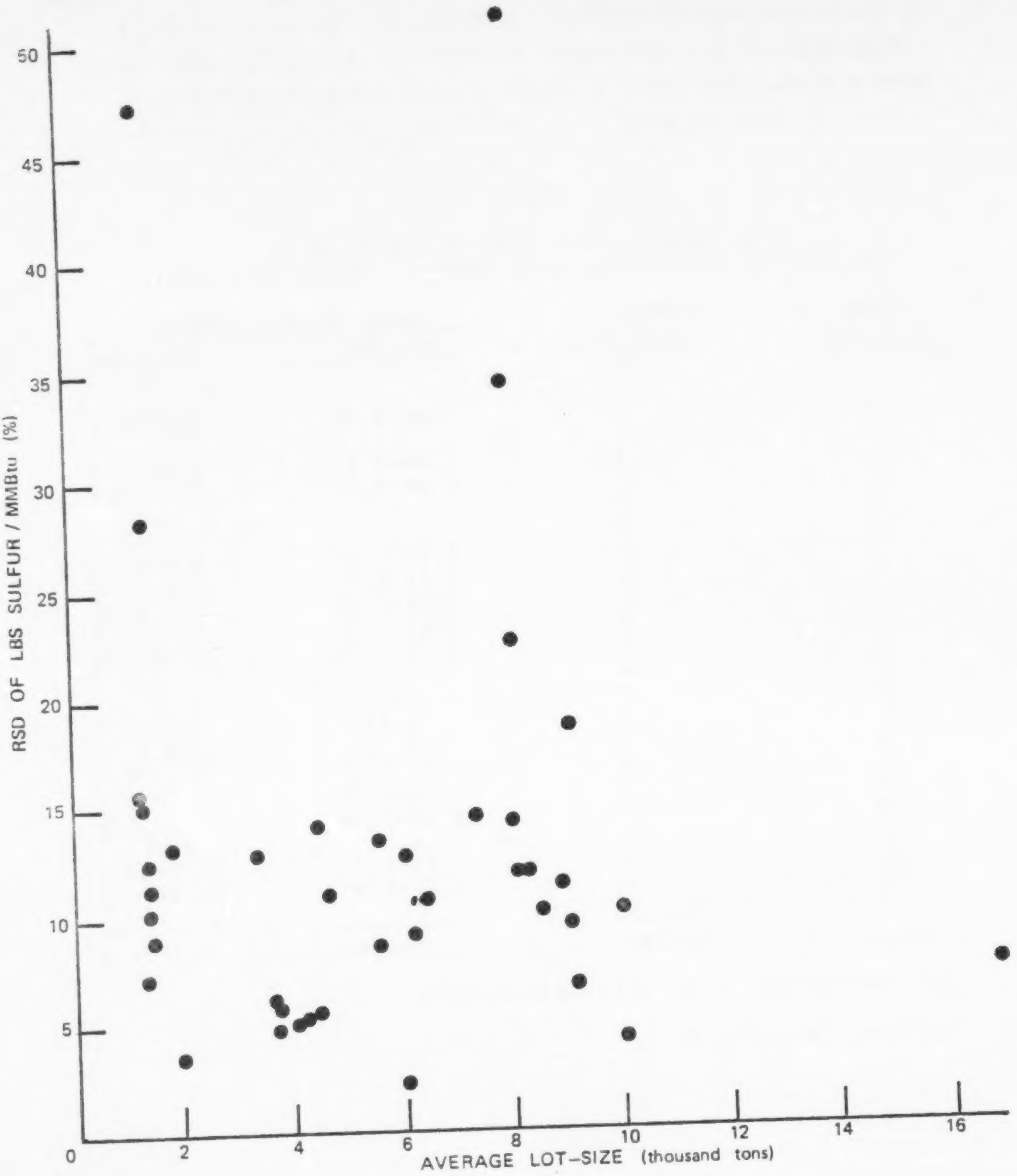
USBM District (1)	Number of Data Sets[1] (2)	Range of Lbs S/MMBtu Average (3)	 RSD (%) (4)
1	7	1.45-2.18	8.8-32.6
2	0	-	-
3	2	1.90-2.14	8.3-11.6
4	35	2.09-4.99	6.6-67.2
6	0	-	-
7	0	-	-
8	21	0.49-2.31	4.9-62.3
9	13	1.60-4.31	4.9-47.2
10	24	1.02-3.52	2.1-51.0
11	4	1.57-3.71	8.3-34.3
12	2	3.18-3.71	18.1-21.5
13	0	-	-
14	0	-	-
15	1	2.63	22.7
16	1	0.19	20.8
17	4	0.35-0.57	6.5-20.7
18	3	0.38-0.73	11.7-45.8
19	17	0.38-1.08	6.9-31.8
20	1	0.90	27.6
21	3	0.82-1.31	25.2-33.3
22	2	0.42-0.85	17.8-30.1
23	0	-	-
Total	140	0.19-4.99	2.1-67.2

[1] Containing 30 or more observations.

Source: Appendix I.

FIGURE 7

RSD OF LBS. S/MMBtu VS.
AVERAGE LOT-SIZE OF INDIVIDUAL DATA SETS FOR
PRODUCING DISTRICTS 9 (Western Kentucky), 10 (Illinios), and 11 (Indiana)



The 41 data sets included in Figure 7 each have 30 or more observations and generally represent washed coals of seams No. 5 and No. 6, which are common to all three states in this producing area. From Figure 7 no relationship between RSD and lot-size is observed. The majority of the RSD's fall in the range of 5 to 18 percent, but appear to be independent of the lot-size.

Additional analyses of these data were performed to examine the relationship of RSD of lbs S/MMBtu versus average lot-size for:

- individual Producing Districts
- individual coal seams
- raw and washed coals

The results of these analyses were also inconclusive in demonstrating any relationship between RSD and lot-size.

4.1.1.2 Multiple Data Sets for Individual Mines

In some cases the data base contained two or more data sets for the same mine. These data sets were examined and are summarized in Table 18. The circumstances resulting in multiple data sets include: (1) deliveries to the same consumer over different time periods, (2) deliveries to different consumers, and (3) analyses of the same coal reported by two different laboratories.

The one case, mine 10045, reflecting the results of a "split-sample" analyzed by two different laboratories, exhibits average lbs S/MMBtu at 3.13 and 3.15 with RSD's of 6.2 and 5.9 percent, respectively. The difference between the average lbs S/MMBtu is less than one percent, while the difference in the RSD's is approximately five percent.

TABLE 18

COMPARISON OF LBS S/MMBTU FOR MINES REPRESENTED BY MULTIPLE DATA SETS

Mine Code (1)	Lbs S/MMBtu Average (2)	Lbs S/MMBtu RSD (%) (3)	Number of Observations (4)
10031[a]	1.78	14.5	268
	1.80	8.8	220
10019[b]	3.39	4.9	114
	2.71	13.4	31
10036[b]	3.48	10.1	107
	2.96	11.8	61
10042[b]	3.49	2.1	147
	3.34	11.9	77
10045[c]	3.13	6.2	214
	3.15	5.9	213
10097[b]	2.78	11.0	339
	3.00	6.7	269
10062[b]	3.18	18.1	55
	3.71	21.5	53
10077[b]	0.88	17.0	169
	1.08	17.2	74
	1.02	24.6	51
10098[b]	0.43	11.7	41
	0.73	45.8	55
10006[b]	0.48	16.9	1,537*
	0.46	14.2	33
	0.43	11.6	49
	0.44	14.4	213
	0.43	11.9	64
	0.42	12.4	1,780
10038[b]	0.57	16.0	140
	0.79	46.1	29
10039[b]	0.65	31.8	30
	0.98	42.6	28
10061[b]	0.37	14.2	47
	0.38	12.0	52

a/ Same consumer, different time periods.
b/ Different consumers.
c/ Different laboratories.

* Core Analyses.

Source: Appendix I.

The data sets for mine 10031 reflect deliveries during two different time periods to the same consumer. The first data set, covering the period January to December 1975, exhibits an average lbs S/MMBtu of 1.78 with an RSD of 14.5 percent. The second data set, for the period November 1977 to September 1978, reveals an average of 1.80 lbs S/MMBtu and an RSD of 8.8 percent. The more recent data set indicates an increase of 1.1 percent in the average and a decrease of 39 percent in the RSD.

The multiple data sets for the 11 remaining mines are based on coal deliveries from the same mine to different consumers. The time periods of the multiple data sets for each mine are not exactly comparable, although most data sets are based on coals analyzed between 1975 and 1978. The differences between the low and high value of the average lbs S/MMBtu for each mine ranged from 2.7 to 69.8 percent, while a similar comparison for the RSD's showed differences ranging from 15.5 to over 466 percent.

A comparison of the RSD's to average lot-sizes for the individual mines containing multiple data sets again failed to demonstrate any consistent relationship.

4.1.1.3 Mine 10006

The data in Table 18 for mine 10006 permit additional comparisons. The data set containing 1,537 analyses represents core analyses of the coal reserves. The four data sets, containing from 33 to 213 observations, are based on deliveries to four separate utilities, while the data set containing 1,780 observations is based on all shipments from the mine during the period November 1977 to October 1978. All data sets, except for the core analyses, are based on unit-train size shipments of approximately 10,000 tons, sampled by an automatic ASTM belt sampler. For convenience, a summary of these data sets is set out in Table 19.

TABLE 19

COMPARISON OF DATA SETS FOR MINE 10006

Type of Data (1)	Date Range (2)	Lbs S/MMBtu Average (3)	Lbs S/MMBtu RSD (%) (4)	Number of Observations (5)
Core	N/A	0.48	16.9	1,537
As shipped	6/76-12/77	0.44	14.4	213
As shipped	12/76- 3/77	0.46	14.2	33
As shipped	1/78-10/78	0.43	11.9	64
As shipped	1/78- 8/78	0.43	11.6	49
As shipped	11/77-10/78	0.42	12.4	1,780
As shipped[1]	6/76-10/78	0.43	13.6	2,199

[1] Includes one data set of 60 observations which was not analyzed separately.

A comparison of the core analysis data (1,537 observations) with all the available data for coal shipments (2,199 observations) shows that the average lbs S/MMBtu as well as the RSD is lower when calculated based on coal shipment data. The average lbs S/MMBtu for the coal shipments is 10.4 percent less than the average indicated by the core analyses, while the RSD is 19.5 percent less.[1]

A comparison of the data by date ranges indicates a decline in the average lbs S/MMBtu and RSD. Coals analyzed during the period June 1976 to December 1977 exhibit an average of 0.44 lbs S/MMBtu and an RSD of 14.4 percent, while comparable data for the period January 1978 to October 1978 show an average of 0.43 lbs S/MMBtu with a RSD of 11.9 percent. Thus, during the period from June 1976 to October 1978, it appears that a slight decline in the lbs S/MMBtu and a reduction in the relative variability of the coal has occurred.

---

[1] Similar results have been reported for other Western coals. For example, a study conducted at the Navajo Plant by the Salt River Project indicated the mean sulfur content of coal shipments was 10 to 20 percent less than the mean indicated by core analyses.

4.1.2 Lot-Size Interval Analysis of Data

The next step in the analysis of data examined the relationship between lot-size and the RSD of lbs S/MMBtu within individual data sets. As noted in the previous discussion, the expected inverse relationship was not observed. It was hypothesized that variations in the size of individual shipments, which in some cases was substantial as indicated in the RSD of the volumes in Appendix I, were masking the relationship between the RSD and lot-size. To test this hypothesis, the individual data sets were analyzed with respect to lot-size intervals within the individual data sets. For example, a data set may have contained analyses based on shipments ranging from 1,000 to 11,000 tons, while the majority of the shipments were in the ranges of 1,000 to 2,000 tons, and 9,000 to 11,000 tons. The tonnage intervals examined within each data set were based upon a visual examination of the frequency distributions and plots. Generally, intervals were selected to include a minimum of 30 observations. These selected lot-size intervals were then analyzed with the computer analytical program to examine the relationship of RSD to lot-size.

Before the results of this analysis are discussed, it is useful to examine the relationship between RSD and lot-size suggested by a theoretical model.

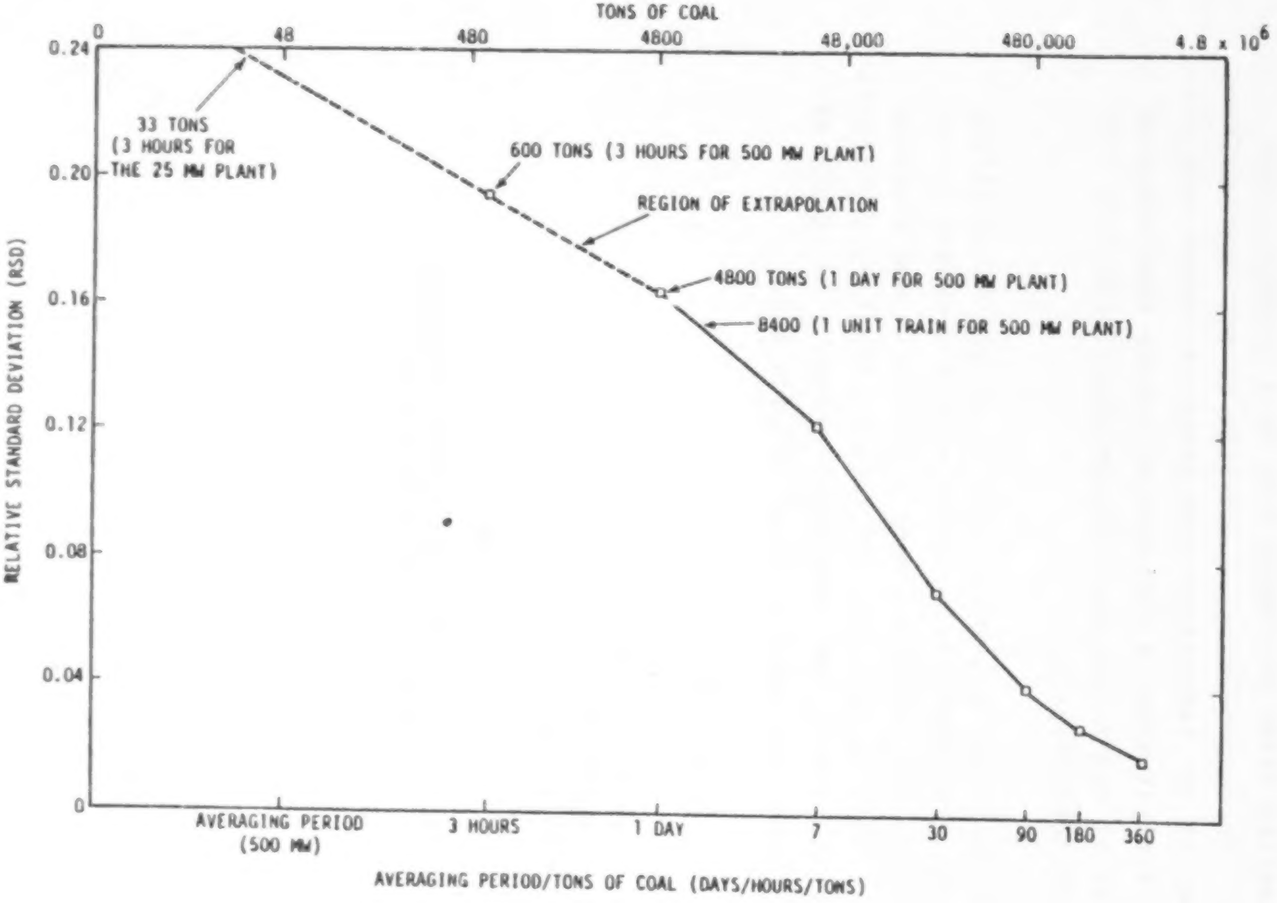
4.1.2.1 Theoretical Relationship Between RSD of Lbs S/MMBtu and Lot-Size

Among the various researchers who have examined the problem of coal sulfur variability, there has been considerable discussion on the theoretical versus the observed relationship between the RSD of lbs S/MMBtu and the lot-size for any particular type of coal. The EPA study "Preliminary Evaluation of Sulfur Variability in Low Sulfur Coals from Selected Mines", while based on limited data, suggests that the RSD increases at a relatively constant rate as the lot-size is successively decreased. Plotted in semilogarithmic form this relationship would approximate a straight line as shown in Figure 8. In contrast, some of the data sets analyzed in Appendix C of the above study displayed a horizontal slope or an upward slope with increasing lot-sizes.

This discussion summarizes the results of a Monte Carlo simulation study which examined the theoretical relationship between lot-size and relative standard deviation. The study concludes that the relationship is approximately linear in semilogarithmic form and the curve has the general shape as shown in Figure 8.

In order to develop this Monte Carlo simulation it was first necessary to postulate a model of the physical process which would determine the relationship between RSD and lot-size. This model assumes that for a small amount of coal, say one ton, there exists a frequency distribution which describes the lbs S/MMBtu as if each ton were used as the basic sampling unit. Under this assumption each time the lot-size is increased, for example to five tons, the value of lbs S/MMBtu is a simple average of five, one-

FIGURE 8



RSD versus averaging period/tons of coal (days/hours/tons).

Source: PEDCo Environmental, "Preliminary Evaluation of Sulfur Variability in Low-Sulfur Coals from Selected Mines", EPA Publication No. EPA-450/3-77-044, July 1977, p.5-8.

ton lot samples drawn at random from this frequency distribution. Correspondingly, a lot-size of ten tons would be the average of ten individual tons drawn at random from the frequency distribution, a lot-size of twenty tons would be the average of twenty one-ton samples drawn from this frequency distribution, etc.

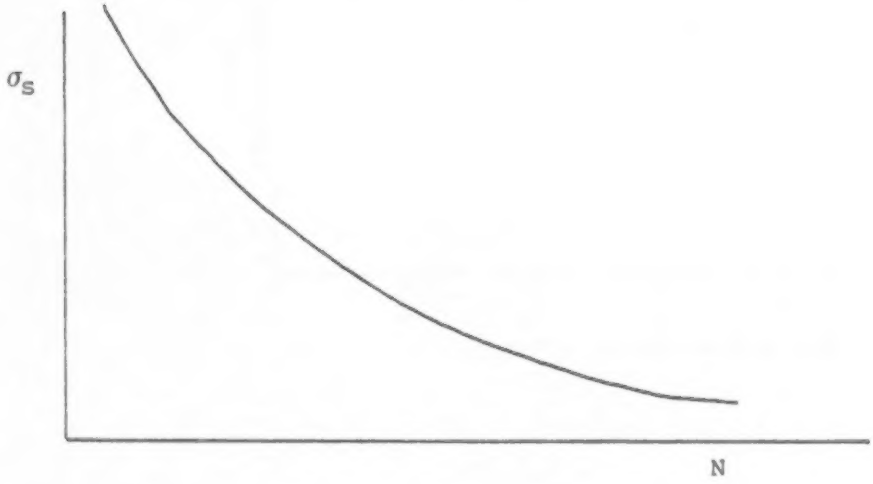
Given any distribution, it can be argued on intuitive grounds that the standard deviation should decrease as the square root of the number of sample points in one ensemble increases. This is given by the formula relating the sample standard deviation to the population standard deviation as the sample size increases as shown in Figure 9.

$$\sigma_s = \frac{\sigma_p}{\sqrt{N}}$$

where σ_p = population SD
σ_s = sample SD
N = sample size

FIGURE 9

RELATIONSHIP BETWEEN SAMPLE STANDARD DEVIATION AND SAMPLE SIZE



By definition, the standard deviation of an ensemble is

$$\sigma_S = \frac{\Sigma(X_i-\bar{X})^2}{N-1} \quad \text{where } \bar{X} = \frac{\sum^{N} X_i}{N}$$

Increasing N, i.e., the lot-size, results in

$$Y_N = \frac{\sum^{N} X_i}{N}$$

and

$$\bar{Y}_N = \frac{\sum^{M} Y_N}{M}$$

The overall variance is given by

$$\sigma^2_{Y_N} = \frac{\sum^{M} (Y_N-\bar{Y}_N)^2}{M}$$

If M is large enough, then

$$\sigma^2_{Y_N} = \sigma^2_P$$

The objective is to find the individual value of $\sigma_S/\bar{X}$, i.e., RSD, of one lot-size, as N increases. Because this problem is very difficult to solve analytically, a simulation approach was used.

The simulation procedure was based on an assumed lognormal distribution, using actual coal analysis data. The data used in the model are based on mine 02020 in Producing District 16 (Colorado). The data for this mine exhibited an average lot-size of 237 tons and a mean of 0.2558 lbs S/MMBtu with a standard deviation of 0.048. The simulation model assumed a lognormal distribution, generated random numbers, and calculated the new means and standard deviations as the lot-size was increased one-fold, two-fold, and so on. The RSD's were then plotted against the resulting lot-sizes.

The results of the simulation study are shown in Figure 10. The horizontal axis on Figure 10 shows the successive lot-sizes, while the vertical axis indicates the relative standard deviation. The figure shows a curve which suggests that the relationship between RSD of lbs S/MMBtu and lot-size is approximately linear in semilogarithmic form, and the relative standard deviation increases greatly as lot-size decreases.

It should be pointed out that this discussion is based on a theoretical model. Although the assumptions used in this model appear to be reasonable, sufficient research has not been conducted to determine whether coal sulfur variability follows the simple physical process model developed in this discussion.

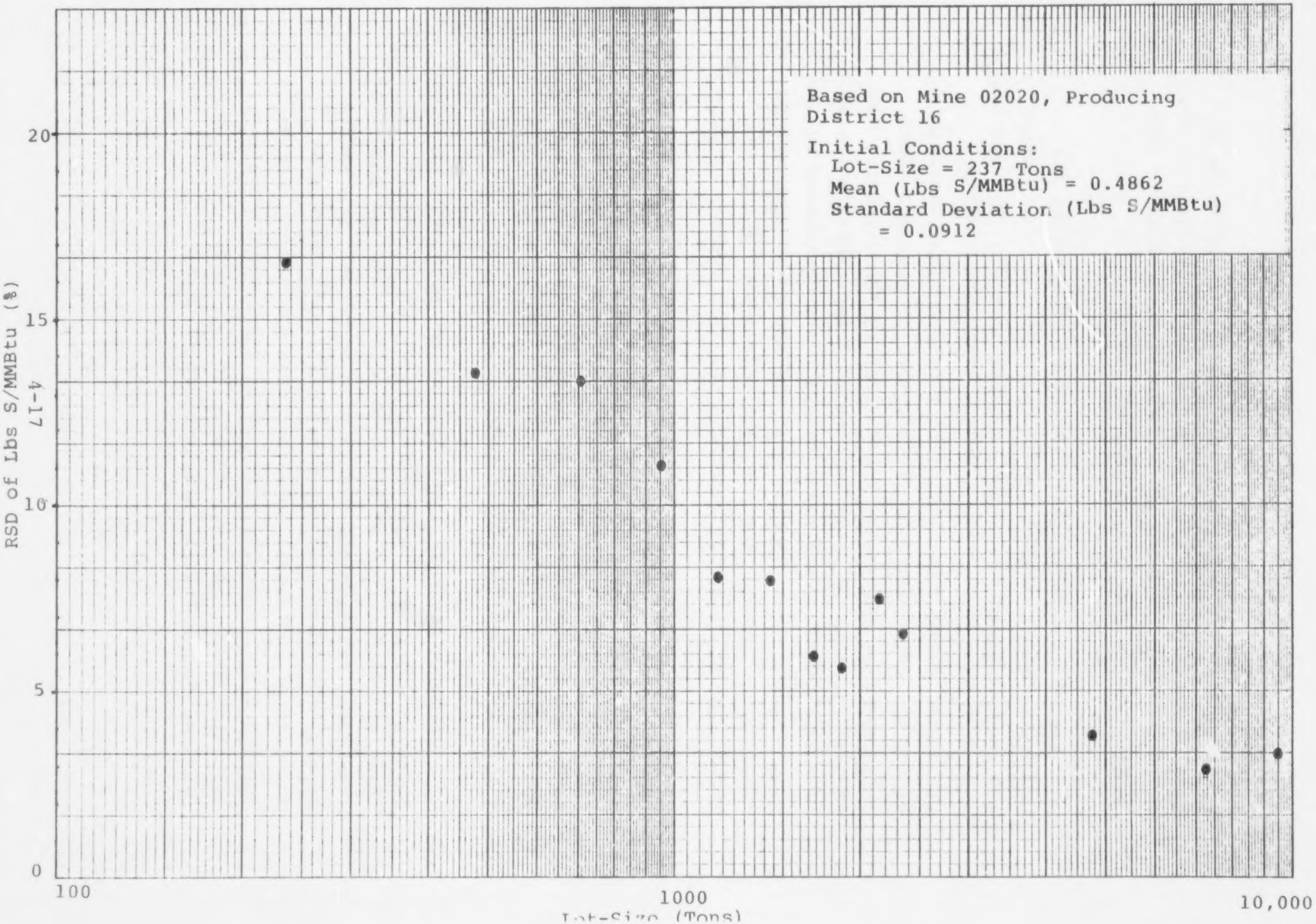
4.1.2.2 Observed Relationship Between RSD of Lbs S/MMBtu and Lot-Size

The purpose of the analysis presented in this section is to determine whether or not the expected theoretical relationship between RSD of lbs S/MMBtu and lot-size is confirmed by coal analysis data.

To begin the analysis, one must first recognize that RSD is a statistical estimate and is subject to sampling error. In order to allow for this variation, the first step in the analysis is to calculate a relative error, the standard deviation, of the RSD estimates. Given this number, one can then discuss not only the absolute value of the

FIGURE 10

RESULTS OF SIMULATION MODEL FOR
RELATIONSHIP OF RSD OF LBS S/MMBTU
AND LOT-SIZE



relative standard deviation, but also limits in terms of plus or minus two, or plus or minus three, standard errors. In this manner the analysis can explicitly recognize the uncertainty in the calculations of RSD's.

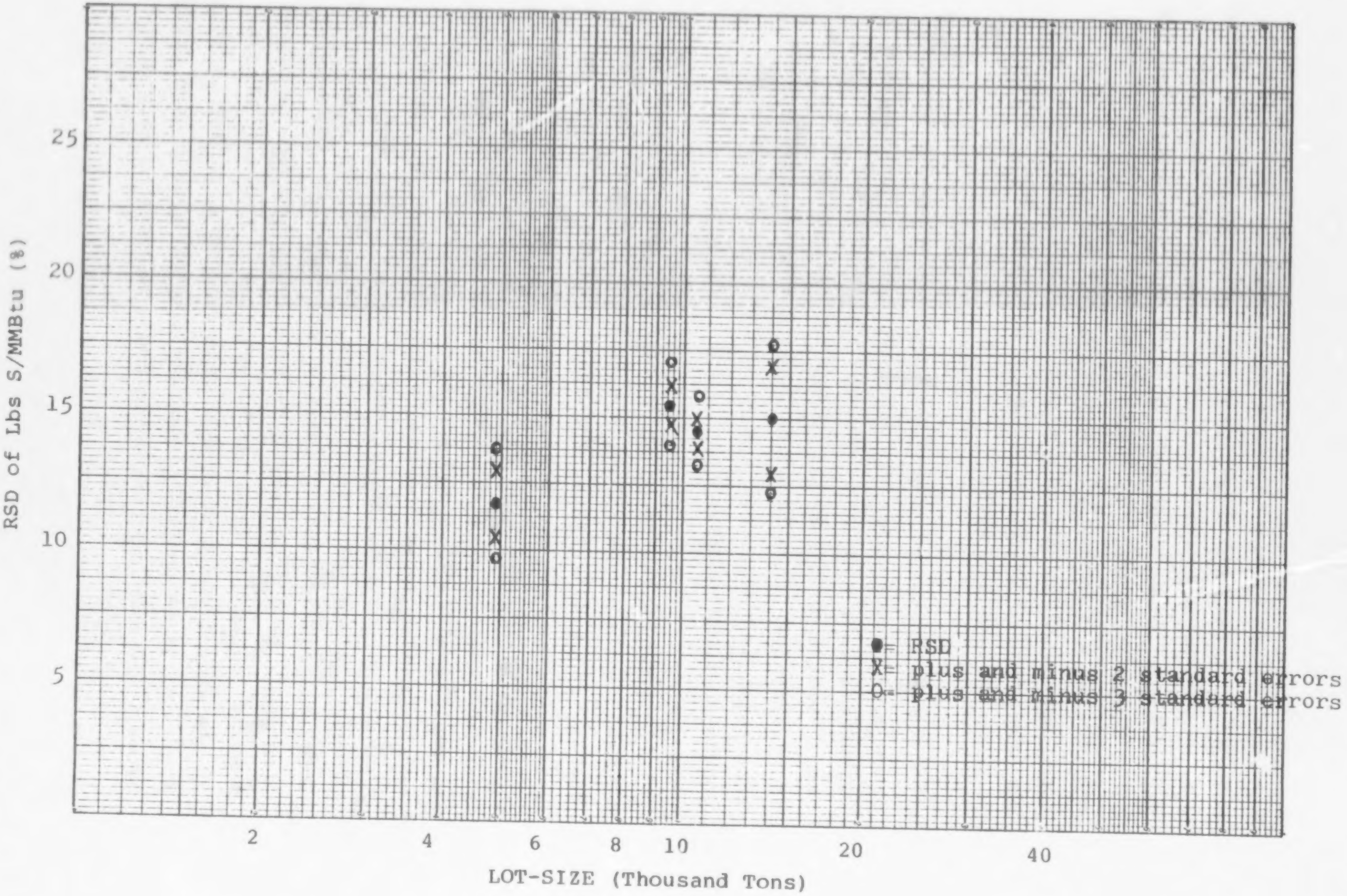
Figure 11 illustrates this method of analysis for a particular mine. The horizontal axis of the figure shows the natural logarithm of the lot-size in tons. The vertical axis shows the RSD of lbs S/MMBtu. The actual RSD's are shown by the dots, while the plus and minus two standard errors are represented by the x's and the plus and minus three standard errors are represented by the circles. Each of the four sets of plots on Figure 11 represents a distinct lot-size interval, the means of which are designated by the dots. Note in this example that the RSD is an increasing function of lot-size for all data points except one, and shows a substantial decrease in RSD with a decreasing lot-size.

A total of 82 mines were analyzed as described in the previous paragraph. Of these, 53 had three or more RSD measurements and 29 had two RSD measurements. Again, each RSD measurement represents a distinct lot-size interval within the individual mine data.

Of the mines with three or more RSD measurements, eight showed a log-linear relationship to within plus or minus three standard errors on all data points, and ten showed a log-linear relationship within plus or minus two standard errors. Of these eighteen mines, RSD was a decreasing function of increasing lot-size (negative slope) in nine cases and an increasing function of increasing lot-size (positive slope) in seven cases. In two cases, the RSD remained constant with increasing lot-sizes.

FIGURE 11

RSD Lbs S/MMBtu Versus Lot-Size
for Mine 10142, Producing District 4



For 35 out of the 53 mines with three or more data points, no consistent log-linear relationship could be shown within plus or minus three errors. However, a nonlinear relationship between RSD and lot-size could be estimated for 20 of these 35 mines. Of these 20 mines, 14 showed RSD to decrease as a function of increasing lot-size, while six cases showed RSD to be an increasing function of lot-size. The remaining 15 mines exhibited a U-shaped relationship between RSD and lot-size.

For 29 of the 83 mines analyzed, only two RSD measurements or lot-size intervals were available. Naturally, a log-linear line will pass through the two points in every case. Twelve of the mines showed RSD decreasing with increasing lot-size, 16 mines showing RSD increasing with increasing lot-size, and 1 mine indicated a relatively constant relationship.

In conclusion, there is no apparent consistent relationship between the RSD of lbs S/MMBtu and lot-size for any individual mine. The coal analysis data examined in this exercise fail to confirm the theoretical relationship which indicated that the RSD of lbs S/MMBtu decreases with increasing lot-sizes. However, other variables, such as geological factors and mining methods, may have a relatively greater impact on coal sulfur variability than lot-size and may have distorted the expected relationship between RSD and lot-size. Further, it should be noted that this analysis is based primarily upon samples representing from 1,000 to 10,000 tons with relatively few points representing volumes of less than 1,000 tons. However, because of the wide dispersion of RSD measurements examined in this analysis, it is unlikely that any simple relationship exists between RSD and lot-size which could be used to accurately predict the RSD for a corresponding lot-size within an individual mine.

These findings are particularly troublesome for coal producers and/or coal consumers who must guarantee that coals will comply with sulfur emission standards. Coal sulfur variability is often critical in the cases of small coal-fired boilers and regulations which specify short averaging periods, both of which would correspond to small lot-sizes. The results of the previous analysis indicate that the short-term sulfur variability of coal from an individual mine cannot be accurately predicted, even if substantial historical data are available. These findings are even more onerous with respect to the development of new mines which must rely on core analysis data. Individual core analyses may represent in excess of 500,000 tons of coal reserves, while potential customers frequently need to know the sulfur variability on increments of 10,000 tons or less. This suggests that the language and requirements of many sulfur dioxide emission-limiting regulations are not consistent with the state of knowledge concerning coal sulfur variability.

4.1.3 Analysis of Data on an Aggregate Basis

The objective of this analysis was to examine the problem of coal sulfur variability within individual Producing Districts. This was accomplished by various aggregations of the data available for each Producing District. In general, and subject to the availability of data, the following analyses were performed on a Producing District basis:

- All coals; with and without lot-size intervals.
- Raw coals; with and without lot-size intervals.
- Washed coals; with and without lot-size intervals.
- Selected coal seams; with and without lot-size intervals.

In addition, data for some Producing Districts permitted more detailed analysis of other factors which have been identified as possible sources of variability in coal analysis data.

The analyses performed and the results obtained from the aggregate analysis are discussed in detail in Appendix J. Although no indisputable conclusions resulted from these aggregate analyses, it was possible to make some general statements about the relationships exhibited in the various Producing Districts.

The results pertaining to the relationship between RSD of lbs sulfur/MMBtu were inconclusive. However, it appears that as the level of aggregation is increased from mine, to seam, to Producing District, the data tend to exhibit a sharp increase in RSD at small lot-sizes, especially below 2,000 tons. This suggests that variables that could not be controlled or were not analyzed may have masked the relationship between RSD and lot-size. In the case of the composite data, it is possible that the effects of these variables tended to cancel each other.

Comparisons of washed and raw coals within the individual Producing Districts consistently indicated a lower lbs S/MMBtu and a lower RSD for the washed coals. These findings tend to support the hypothesis that coal washing would, in general, reduce the level of sulfur emissions as well as the relative variability of the emissions.

In the limited number of cases analyzed, it appears that significant differences in RSD's can exist among seams within the same Producing District. As reported in previous sections of this report, substantial, inconsistent differences in RSD's were also observed among individual mines and among lot-sizes within mines. These observations raise serious doubts about the extent of the relationship between RSD and lot-size and the existence of a simple relationship which can accurately generalize coal sulfur variabilities.

Finally, some of the data on an aggregate basis exhibited large RSD's, frequently in excess of 40 percent. RSD's of this magnitude could have a substantial impact on compliance. This suggests that coal consumers subject to a given emission limit with only marginally acceptable coals, must selectively evaluate the various sources of supply and may consequently find it necessary to exclude those sources which exhibit large variabilities.

4.2 Predictive Ability of Producing District and Seam Data for Individual Mines

This section shows the results of a study performed to determine whether composite seam or Producing District data can be used to predict the relative variability of lbs S/MMBtu for an individual mine. If combined data can predict mine data, the derived relationships between RSD and lot-size for composite seams or Producing Districts would permit the estimation of RSD's for an individual mine over a range of lot-sizes.

Generally, the individual mine data sets permitted the calculation of the relative standard deviation for only several lot-sizes. However, by combining the data within

Producing Districts or seams, it was possible to calculate RSD's for a wide range of lot-sizes.

4.2.1 Methodology

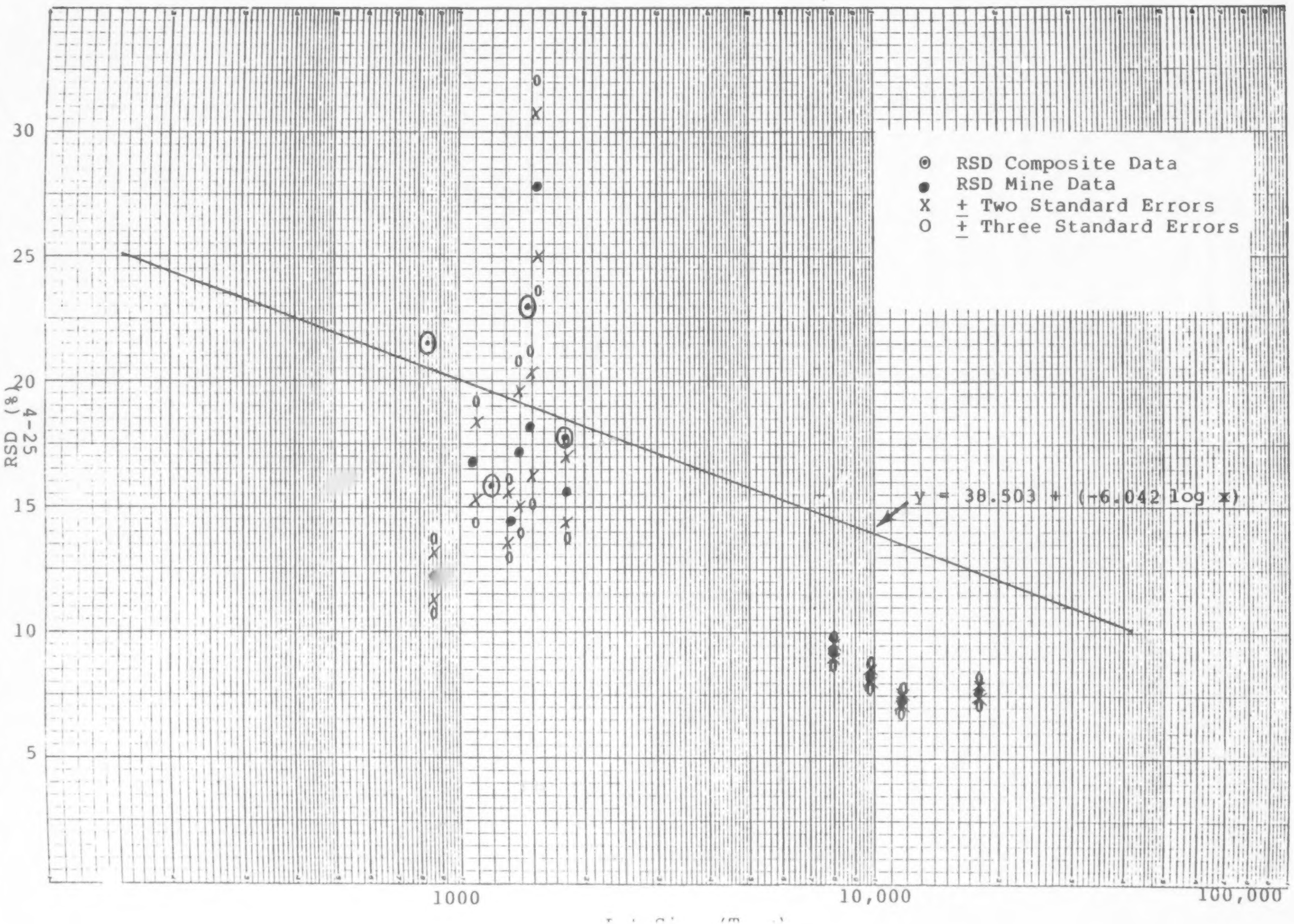
Data from all mines within a seam or district were combined and analyzed by lot-size in terms of the relative variability of lbs S/MMBtu. A line of regression was then fitted between the log of the lot-size and RSD and used as the predictor of the RSD of lbs S/MMBtu within individual mines. Standard errors of the RSD estimates for individual mines were then calculated. The individual mine RSD estimates, as well as the composite seam data, were then plotted on the same graph. Two tests were used to determine the degree to which the seam data could predict mine RSD. First, mine RSD estimates were checked against the RSD as predicted from the composite data to determine if the regression line fell within plus or minus three standard errors of the individual mine RSD's. Second, the absolute average error was analyzed by assuming the composite data were predictors of mine RSD's.

4.2.2 Analysis of Predictive Capabilities of Seam Data

Figure 12 shows the results of this analysis for the composite seam data for the Pittsburgh seam (036), Producing District 4 (Ohio). The regression line intersected only two of the eleven mine RSD's within plus or minus three standard errors. Of the remaining nine mine RSD estimates, the error ranged from a high of 8.75 percentage points to a low of 3 percentage points with an average of 5.6. This is a substantial error since the RSD's for individual mines generally ranged from 15 to 25 percent. Furthermore, note

Figure 12

Analysis of Composite Seam Data
For the Pittsburgh Seam (036)
in Producing District 4



that the composite data were biased estimators of mine RSD's. All except one of the mine RSD's lie below the composite regression line.

Similar analyses were performed for the Middle Kittanning (080) and Lower Kittanning (084) seams in Producing District 4. For the Middle Kittanning seam, there were 27 individual mine RSD estimates, of which only 6 fell within the plus or minus three standard errors. The average error in this case ranged from 8.7 percentage points to a low of 4 percentage points, with an absolute average error of 8.7 percentage points. For the Lower Kittanning seam, 6 of the 14 individual mine RSD estimates were within plus or minus three standard errors of the composite RSD. The absolute average error for this seam ranged between a high of 10 percentage points to a low of 1.75 percentage points, with an average of 3.8. In both instances, the composite data provided biased estimates which consistently over-estimated mine RSD's.

In addition to the three seams discussed above, several other seams were plotted and the composite mine data were compared to the individual mine estimates. These cases exhibited the same general results as previously discussed, that is, the composite data were not a good predictor of mine RSD's.

Similar analyses were attempted using variance rather than RSD as a measure of variability. In these instances, the results were the same. Composite data were not a good predictor of mine variability.

4.2.3 Analysis of Predictive Capabilities of Producing District Data

Figure 13 shows the same type of analysis for composite data for Producing District 4 (Ohio). Analysis of this data provides the same general conclusion as the composite seam data. The estimates are clearly biased in that the composite Producing District data overestimate mine RSD's in nearly all instances. The composite Producing District RSD estimate falls within plus or minus three standard errors for only 6 individual mine RSD estimates. Furthermore, the absolute error is fairly large and because of the dispersion of mine RSD's, there is no curve which will yield a reasonable estimate of RSD.

Analyses with composite data were also performed for Districts 01, 08, 10, and 11. The results were the same as previously discussed. The composite Producing District data were a biased and inaccurate estimate of mine RSD's.

As in the case of the composite seam analyses, separate analyses were performed using variance instead of RSD. Again, there were no significant differences in the results.

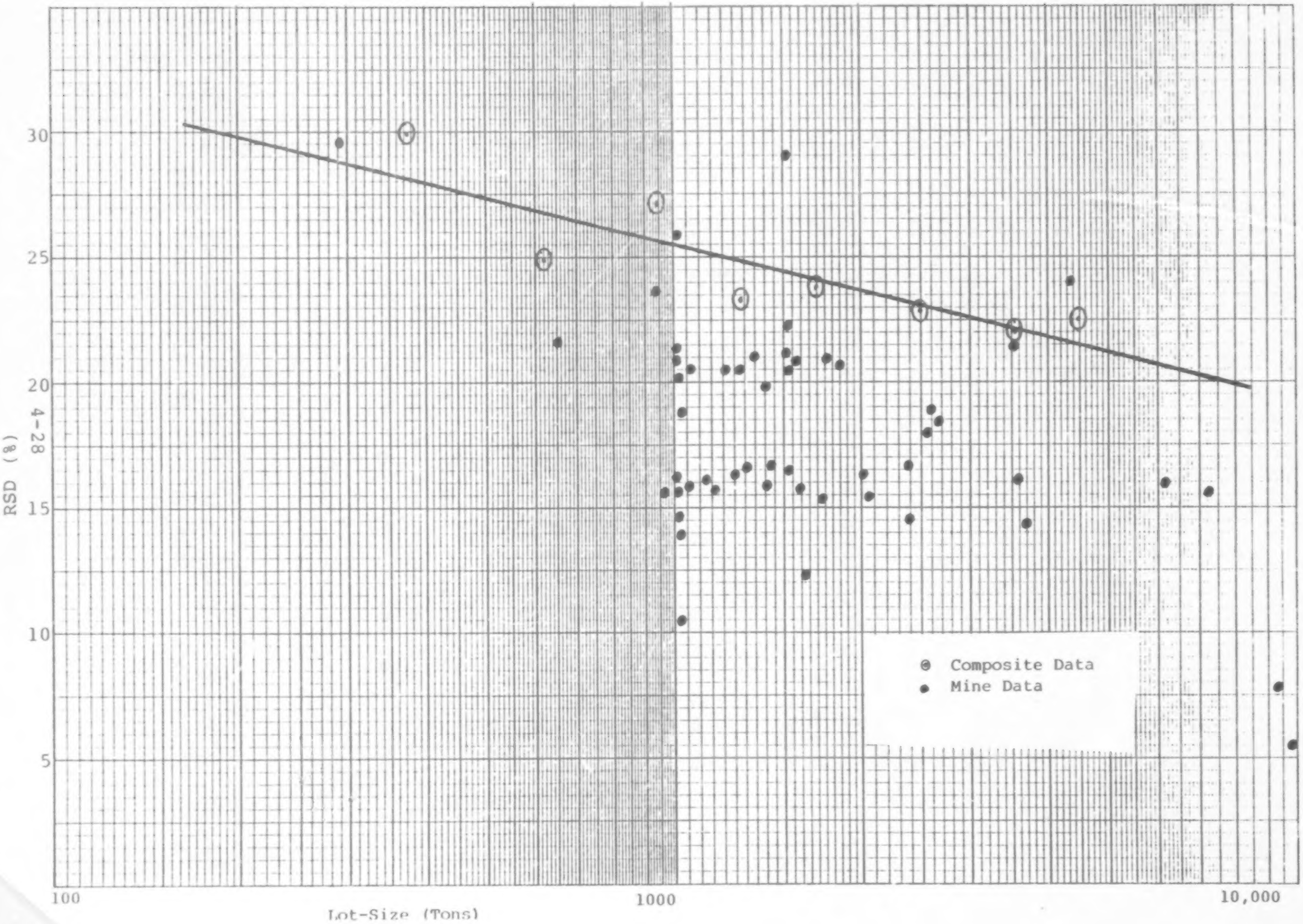
In conclusion, composite seam or composite Producing District data cannot be used to accurately predict the variability of lbs S/MMBtu for individual mines.

4.3 Analysis of the Statistical Distributions of Coal Characteristics

This section examines the coal analysis data provided by the respondents with respect to frequency distributions. As indicated in Section 2.4, the differences between the

Figure 13

ANALYSIS OF COMPOSITE PRODUCING
DISTRICT 4 DATA AND DATA FOR
INDIVIDUAL MINES (RAW COALS)



three distributions examined -- normal, lognormal, and inverted gamma -- can have a significant impact on compliance with sulfur emission regulations.

Most statistical analyses of coal variability assume drawing at random from a particular type of frequency distribution. This section attempts to provide recommendations as to the most appropriate frequency distributions, based on the observed frequency distributions of actual coal data. In this analysis the frequency distributions of sulfur and heat contents, as well as lbs S/MMBtu, were examined.

The "best" distribution depends upon the specific purposes or requirements of the analysis. This section analyzes three possible alternative requirements. The first is to most accurately predict the top 1.5 percent, or the extreme right tail, of the distribution. The second requirement examined is to predict the top 15 percent of the right tail of the distribution. The final requirement examined is for the prediction of the total distribution. Separate methods of analysis were chosen for each of these three criteria.

4.3.1 Methods of Analysis

Table 20 presents an example of the analysis performed to determine which frequency distribution best fits the observed data in the top 15 percent of the distribution. The cell information in Table 20 refers to the cells used in the computer analysis program, described in Section 3 of this report. The sum of cells 7, 8, and 9 is approximately equal to the top 15 percent of the distribution. The relative error is the observed frequency minus the expected frequency divided by the expected frequency. Since the number of observations in the top three cells can be rather

TABLE 20

ANALYSIS OF THE TOP 15 PERCENT OF THE FREQUENCY DISTRIBUTIONS BY COMPUTATION OF RELATIVE DISTRIBUTION ERROR[1/]

Cell Number (1)	Number of Data Points: Observed (2)	Number of Data Points: Expected (3)	Relative Error (%) (4)
Analysis of Heat Content			
Normal			
9	0	.1	--
8	2	2.4	-16.66
7	10	24.1	-58.51
9,8,&7	12	26.6	-54.89
Inverted Gamma			
9	0	.2	--
8	2	3.0	-33.33
7	10	24.5	-59.18
9,8&7	12	27.7	-56.68
Lognormal			
9	0	.1	--
8	1	2.4	--
7	9	24.1	-62.66
9,8,&7	10	26.6	-62.41
Analysis of Lbs Sulfur/MMBtu			
Normal			
9	3	.1	2900.00
8	8	2.4	233.33
7	13	24.1	-46.06
9,8,&7	24	26.6	- 9.77
Inverted Gamma			
9	3	1.0	200.00
8	8	5.4	48.15
7	13	24.3	-46.50
9,8&7	24	30.7	-21.82
Lognormal			
9	3	.1	2900.00
8	6	2.4	150.00
7	15	24.1	-37.76
9,8,&7	24	26.6	- 9.77

[1/] For mines with more than 200 observations and cells containing more than 2 observations.
Relative Error = $\frac{\text{observed - expected}}{\text{expected}}$

Source: Foster Associates, Inc.

small, the relative error can be fairly large without indicating a poor fit.

The method for analyzing the best fit for the top 1.5 percent of the distribution is shown in Figure 14. This figure is based on the same data presented in Table 20. If the observed and expected frequencies agreed exactly, the lines for all three distributions would lie along the 45 degree axis. In this example the inverted gamma distribution gives a much more accurate fit than either the lognormal or the normal.[1]

The method used to analyze the overall goodness of fit was the chi-square test. The chi-square statistic is determined by the following formula:

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Where O_i = observed frequency in cell i

E_i = expected frequency in cell i

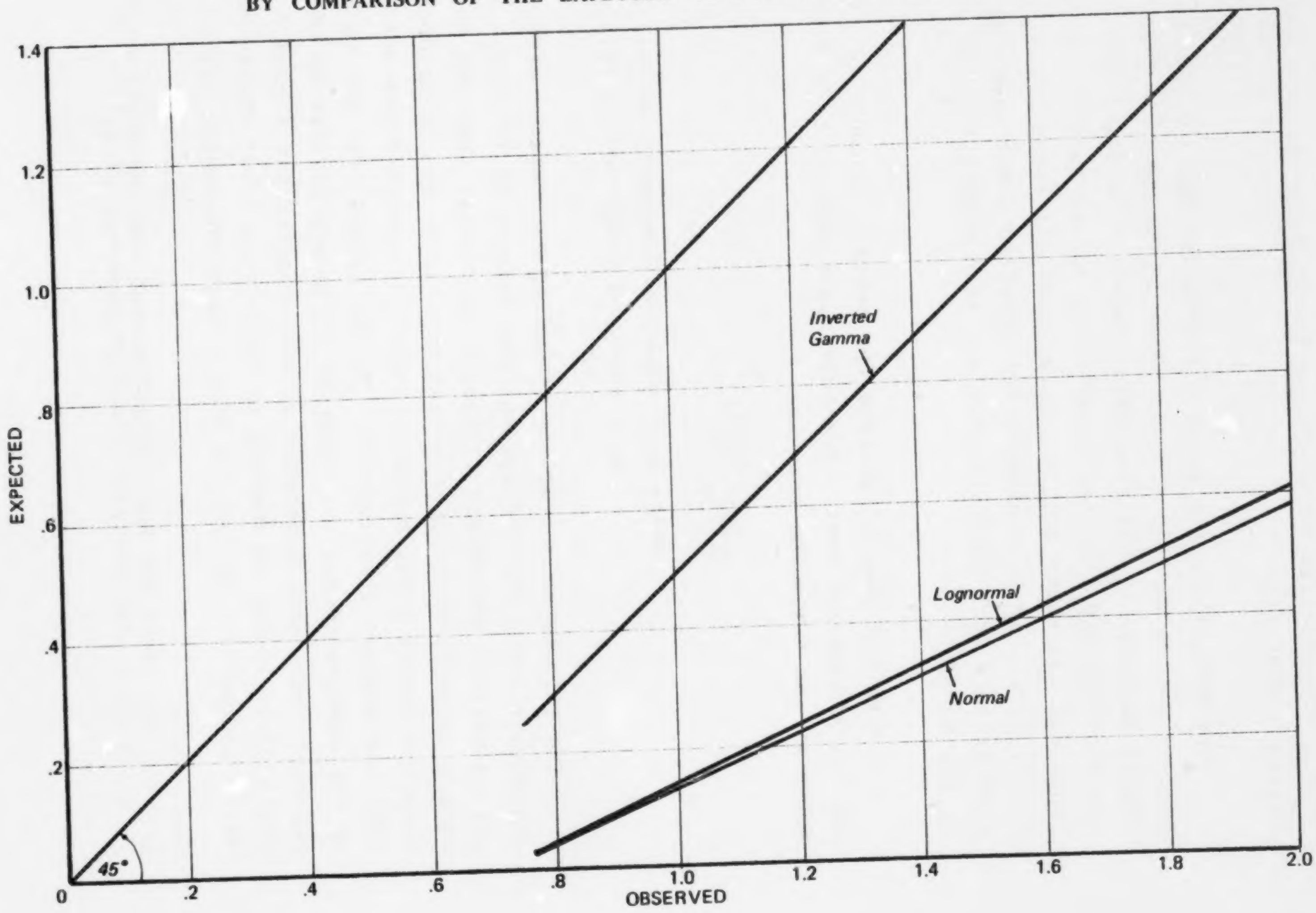
The calculated chi-square statistic is compared with a chi-square value for the appropriate degrees of freedom. If the calculated chi-square statistic is greater than the chi-square value from the appropriate tables, it can be assumed that the actual distribution of the coal variable was not drawn from the assumed distribution. To the extent that the square of the observed minus the expected frequency divided by the expected represents a measure of what constitutes a good fit, for a given number of degrees of freedom, a lower chi-square value indicates a better fit than a high chi-square value.

---

[1] The data sets did not contain enough extreme observations to allow more sophisticated tests of goodness of fit.

FIGURE 14

ANALYSIS OF THE TOP 1.5 PERCENT OF THE FREQUENCY DISTRIBUTIONS BY COMPARISON OF THE EXPECTED VS OBSERVED FREQUENCIES (%)



4.3.2 Results of Best Fit Analysis

The results of the analysis of the statistical distributions of coal characteristics are discussed in detail in Appendix K of this report and are summarized in Table 21. Column (1) in the table identifies the coal characteristic analyzed, while Column (2) differentiates the raw and washed coals. Column (3) identifies the portion of the frequency distribution for which the analysis was performed -- top 1.5 percent, top 15 percent, or the total distribution. Columns (4), (5), and (6) note the number of data sets which were best represented by the respective normal, inverted gamma, and lognormal distributions. It should be noted that the data sets represented in Table 21 were not sorted by lot-size. Lot-size analyses of these data sets are discussed in Appendix K.

In general, no firm conclusions could be made with respect to the best distribution for the coal characteristics analyzed. The distributions exhibiting the best fit varied considerably from data set to data set. However, the inverted gamma distribution appears to provide the best fit for lbs S/MMBtu, while heat content is best represented by the normal distribution. In the limited number of data sets analyzed for sulfur content, supplemented by visual examination of numerous distributions, the inverted gamma distribution provided the best fit.

A comparison of the results for raw and washed coals indicated that coal washing does not alter the shape of the distribution for lbs S/MMBtu or heat content. However, as previously reported, coal washing appears to alter the mean of the distribution and reduce the relative variability of the characteristics investigated.

In view of the results of these analyses it seems appropriate to offer the following recommendations for both raw and washed coals:

Coal Characteristic	Best Fit Distribution
Lbs S/MMBtu	Inverted Gamma
Heat Content (Btu/Lb)	Normal
Sulfur Content	Inverted Gamma

TABLE 21

SUMMARY OF BEST FIT ANALYSIS FOR THE OBSERVED FREQUENCY DISTRIBUTIONS OF COAL CHARACTERISTICS1/

			Number of Data Sets Best Fit By:		
Coal Characteristic (1)	Type of Coal (2)	Portion of Distribution (3)	Normal (4)	Inverted Gamma (5)	Log-normal (6)
Lbs S/MMBtu	Raw	Top 1.5 Percent	4	16	19
	Raw	Top 15 Percent	23.5	10	7.5
	Raw	Total	14	15	14
	Washed	Top 1.5 Percent	3	7	5
	Washed	Top 15 Percent	9.5	4	3.5
	Washed	Total	1	5	4
Heat Content	Raw	Top 1.5 Percent	13	10	5
	Raw	Top 15 Percent	33	5	2
	Raw	Total	17	9	2
	Washed	Top 1.5 Percent	9	2	3
	Washed	Top 15 Percent	12	4	1
	Washed	Total	5	4	4
Sulfur Content	Raw	Total	15	17	8
	Washed	Total	1	3	5

1/ Data sets not sorted by lot-size.

Source: Appendix K.

5.0 Analysis of Bureau of Mines Data

This section summarizes the results of the analyses performed on data contained on the Bureau of Mines Coal History Data Tape ("detail tape"). The coal samples and analyses contained on this tape were collected by the Bureau of Mines during the period from 1966 to 1977 and reflect coals purchased by various Government agencies. The purpose of these analyses is to determine whether coal suppliers are providing coal of the specifications defined in coal purchase contracts. The samples are analyzed by the Bureau of Mines laboratory and if the coal is not of the quality quaranteed by the contractor, price adjustments are made.

Although the "detail tape" contains both tipple samples, collected after the coal has received final treatment at the tipple or cleaning plant, and delivered samples, collected during coal unloading at the destination, only the delivered samples were analyzed for variability.

All samples were collected in accordance with instructions issued by the Bureau of Mines.1/ The individual increments or "cuts" represent complete cross-sections of the entire stream of coal, taken regularly throughout the period of unloading, so that all parts of the shipment are equally represented. These individual increments comprise the gross sample, which weighed not less than 1,000 pounds. In addition, the maximum tonnage represented by one gross sample was generally limited to 1,000 tons.

Time and cost constraints prohibited an analysis of all the data contained on the "detail tape." Only selected seams

---

1/ Snyder, N. H. (Rev. by S. J. Aresco), Coal Sampling Revision to Technical Paper 133. Bureau of Mines Handbook, 1957.

and mines were analyzed, as discussed below.

5.1 Analysis of Selected Coal Seams

The criteria for selection of the coals seams analyzed were primarily based on the major commercial steam-coal seams in the United States as identified by Averitt.[1] These are the few, thick seams, which are continuous over large areas and possess special properties which make them commercially desirable. These seams contain a substantial portion of the domestic coal reserve base, and they have yielded the bulk of past production.

Although data contained in the "detail tape" were not sufficient to analyze all of the major commercial steam-coal seams, it was possible to analyze representative seams for the Northern and Southern Appalachian, Mid-Continent and Western producing areas. The results of these analyses are set out in Table 22 and are summarized below by producing area. It should be noted that all of these analyses were performed on a lot-size basis. The intervals examined were 0 to 300 tons, 300 to 600 tons, and 600 to 1,000 tons, the midpoints of which are set out in Column (4) of Table 22.

5.1.1 Northern Appalachian

Two coal seams were analyzed in the Northern Appalachian area -- the Pittsburgh seam (036) and Lower Kittanning seam (084). Data for the Pittsburgh seam indicated an inverse relationship between RSD and lot-size; however, data for this seam were limited to less than 30 observations per lot-size interval.

---

[1] Averitt, Paul, Coal Resources of the United States, January 1, 1974, U.S. Geological Survey Bulletin 1412, 1975.

TABLE 22

ANALYSIS OF COAL SEAMS BY LOT-SIZE (USBM "DETAIL TAPE" DATA)

Producing Area (1)	Seam Code/Name (2)	Preparation (3)	Midpoint of Lot-Size (Tons)1/ (4)	Lbs S/MMBtu		Number of Analyses (7)
				Average (5)	RSD (%) (6)	
Northern Appalachian	036 Pittsburgh	Raw	128	2.38	39.56	19
			427	2.40	34.18	18
			971	2.71	13.73	8
Northern Appalachian	084 Lower Kittanning	Raw	186	1.96	48.74	96
			449	1.64	29.51	94
			857	1.68	13.56	229
Southern Appalachian	151 Upper Elkhorn #3	Raw	172	0.67	40.97	102
			443	0.59	27.47	96
			901	0.68	35.28	431
Southern Appalachian	299 Black Creek	Raw	167	0.76	47.43	53
			443	0.79	53.37	41
			970	1.00	48.93	619
Mid-Continent	484 Herrin #6	Washed	146	2.29	25.27	493
			464	2.35	23.45	396
			909	2.24	19.49	2444
Western	846 Hiawatha	Raw	89	0.47	22.19	345
			431	0.43	15.91	24
			834	0.41	24.41	48
Western	750 Wadge	Raw	165	0.63	19.50	108
			428	0.58	14.89	57
			844	0.47	13.07	38

1/ The intervals specified for each seam were:

0 - 300 tons
300 - 600 tons
600 - 1000 tons

Column (4) identifies the midpoint of the intervals for each data set.

The Lower Kittanning seam (084) also exhibited the inverse relationship between RSD of lbs S/MMBtu and lot-size. In this case, the RSD declines from 48.7 percent at the 0 to 300 ton interval to 13.6 percent at the 600 to 1,000 ton interval.

5.1.2 Southern Appalachian

The Upper Elkhorn #3 (151) and Black Creek (299) seams were analyzed in the Southern Appalachian producing area. Data for these seams do not exhibit an inverse relationship between RSD and lot-size. As in the case of the Northern Appalachian coals, these coals appear to have rather large RSD's for the 0 to 300 ton lot-size. However, the Northern Appalachian seams exhibited a progressive decline in RSD with an increase in lot-size, while the RSD's of the Southern Appalachian coals remained relatively constant with respect to lot-size.

5.1.3 Mid-Continent

One coal seam, the Herrin #6 (484), was analyzed in the Mid-Continent producing area. More than 3,000 observations for this seam were available in Bureau of Mines "detail tape." The analysis shows a decline in RSD from 25.3 percent at the 0 to 300 ton interval to 19.5 percent at the 600 to 1,000 ton interval.

5.1.4 Western

Two coal seams, Hiawatha (846) and Wadge (750), were analyzed in the Western producing area. Although the Wadge seam exhibited a moderate decline in RSD with respect to increasing lot-sizes, no such relationship was observed for the Hiawatha seam. In order of magnitude, the RSD's of the smallest lot-sizes (0-300 tons) for the Western producing area seams were slightly less than the RSD for the Herrin #6 seam in the Mid-Continent producing area and substantially less than the RSD's for the Northern and Southern Appalachian areas.

5.1.5 Comparison of Analysis of Seam Data from Bureau of Mines Data with Data Received from Respondents

As was previously discussed, the Bureau of Mines data were based exclusively on lot-sizes less than 1,000 tons each, while data from the respondents generally reflected larger lot-sizes. Data for four seams -- Pittsburgh, Lower Kittanning, Upper Elkhorn #3, and Illinois #6 (Herrin) -- were available from the Bureau of Mines "detail tape" as well as from data received from respondents. A comparison of these data on a lot-size basis is set out in Table 23. In general, Table 23 indicates that when the data from the Bureau of Mines and respondents are compared as a composite, the inverse relationships between RSD and lot-size which may have previously existed, are no longer readily apparent.

5.2 Analysis of Selected Mines

In this analysis 16 individual mines contained within the Bureau of Mines "detail tape" were analyzed. The criteria for the selection of these mines included: (1) mines producing individual coal seams for which a large number of analyses were available, (2) mines for Producing Districts not represented in the data base assembled from respondents' data, and (3) mines for which corresponding data were available from respondents' data, which would be of interest for comparative purposes. The results of this analysis on a

TABLE 23

COMPARISON OF ANALYSIS OF SEAM DATA FROM BUREAU OF MINES WITH DATA RECEIVED FROM RESPONDENTS

		RSD of Lbs S/MMBtu (%)		
Seam (Code/Name) (1)	Midpoint of Lot-Size (Tons) (2)	USBM Data (3)	Data from Respondents (4)	Number of Observations (5)
036/Pittsburgh	128	39.6	--	19
	427	34.2	--	18
	821	--	22.0	15
	971	13.7	--	8
	1,170	--	16.1	42
	1,448	--	23.3	78
	1,782	--	17.8	47
084/Lower Kittanning	186	48.7	--	96
	440	--	20.7	29
	449	29.5	--	94
	857	13.6	--	229
	1,015	--	20.2	105
	1,208	--	20.4	188
	1,407	--	19.9	517
	1,582	--	17.2	91
	2,470	--	18.3	22
151/Upper Elkhorn #3	173	41.0	--	102
	443	27.5	--	96
	901	35.3	--	431
	1,110	--	60.7	64
	1,737	--	30.9	32
	2,812	--	22.9	62
484/Illinois #6 (Herrin)	146	25.3	--	493
	464	23.5	--	396
	909	19.5	--	2,444
	1,388	--	28.8	993
	2,354	--	13.6	172
	3,971	--	16.6	72
	5,897	--	14.3	186
	7,262	--	43.1	155
	8,369	--	45.6	461
	9,581	--	45.3	204

Source: Foster Associates, Inc.

lot-size basis are set out in Table 24 and are briefly discussed below.

5.2.1 Mine 01200, Upper Freeport Seam

Coals analyzed for this mine were washed coals of two distinct sizes -- double screened stoker and single screened slack. The average lbs S/MMBtu for the single screened slack coals were consistently greater than the double screened stoker coals. The RSD's of the lbs S/MMBtu for the single screened coals exhibited a decline for each successive increase in lot-szie. However, this relationship was not observed for the double screened coals.

5.2.2 Mine 00950, Pittsburgh Seam

All coals analyzed were washed, double screened stoker coals. Data for this mine exhibited a direct relationship between RSD and lot-size.

5.2.3 Mine 07290, Middle Kittanning Seam

Data were available only for the 600 to 1,000 ton lot-size for washed coals. At this lot-size the double-screened stoker coals exhibited an RSD of 12.3 percent compared to 10.9 percent for the single screened coals.

Limited data for this mine obtained from an electric utility indicated an RSD of 12.1 percent for shipments averaging approximately 1,000 tons each.

Sheet 1 of 2

TABLE 24

ANALYSIS OF INDIVIDUAL MINE DATA FROM USBM "DETAIL" COAL DATA TAPE

USBM District (1)	Mine Code (2)	Seam (Code/Name) (3)	Preparation (4)	Average of Lot-Size1/ (5)	Lbs S/MMBtu Average (6)	Lbs S/MMBtu RSD (%) (7)	Number of Analyses (8)	Coal Size (9)
1	01200	071/U. Freeport	Washed	188	0.89	11.60	83	Double Screened Stoker
			Washed	425	0.90	15.21	150	Double Screened Stoker
			Washed	869	0.90	12.91	133	Double Screened Stoker
			Washed	196	0.96	13.98	64	Single Screened Slack
			Washed	424	0.97	13.11	73	Single Screened Slack
			Washed	924	1.01	12.63	247	Single Screened Slack
2	00950	036/Pittsburgh	Washed	192	1.37	15.59	29	Double Screened Stoker
			Washed	425	1.38	16.84	41	Double Screened Stoker
			Washed	789	1.25	18.29	55	Double Screened Stoker
4	07290	081/M. Kittanning	Washed	962	2.12	12.31	136	Double Screened Stoker
			Washed	921	2.12	10.87	371	Single Screened Slack
7	00614	100/Various	Raw	950	0.87	18.65	106	Crushed Run-of-Mine
			Washed	957	0.76	18.01	532	Single Screened Slack
8	00637	956/Elkhorn #3 and Hazard #4	Washed	172	0.58	14.86	61	Double Screened Stoker
			Washed	457	0.57	15.24	82	Double Screened Stoker
			Washed	868	0.55	13.52	161	Double Screened Stoker
			Washed	191	0.57	13.50	61	Single Screened Slack
			Washed	451	0.59	12.34	91	Single Screened Slack
			Washed	893	0.56	13.76	355	Single Screened Slack
8	02557	151/Upper Elkhorn #3	Washed	140	0.73	13.96	84	Double Screened Stoker
			Washed	472	0.72	19.69	84	Double Screened Stoker
			Washed	814	0.69	21.88	59	Double Screened Stoker
			Washed	93	0.82	11.89	53	Single Screened Slack
			Washed	398	0.84	23.63	9	Single Screened Slack
			Washed	902	0.78	18.79	113	Single Screened Slack
8	04184	111/Hazard #5-A	Raw	471	0.49	18.28	396	Crushed Run-of-Mine
			Raw	817	0.47	17.14	86	Crushed Run-of-Mine

Sheet 2 of 2

TABLE 24

USBM District (1)	Mine Code (2)	Seam (Code/Name) (3)	Preparation (4)	Average of Lot-Size1/ (5)	Lbs S/MMBtu		Number of Analyses (8)	Coal Size (9)
					Average (6)	RSD (%) (7)		
8	04204	111/Hazard #5-A	Washed	166	0.48	10.73	51	Double Screened Stoker
			Washed	446	0.48	8.60	39	Double Screened Stoker
			Washed	886	0.44	12.73	211	Double Screened Stoker
			Washed	236	0.47	14.09	5	Single Screened Slack
			Washed	480	0.55	10.23	138	Single Screened Slack
			Washed	822	0.59	13.11	126	Single Screened Slack
10	04978	484/Herrin #6	Raw	869	1.77	13.17	87	Crushed Run-of-Mine
			Washed	896	1.64	9.32	301	Single Screened Slack
10	07310	496/Morris #2	Washed	959	2.36	10.48	162	Double Screened Stoker
11	04730	001/#5, #6, #7	Washed	239	0.99	32.35	14	Double Screened Stoker
			Washed	458	1.25	34.53	41	Double Screened Stoker
			Washed	863	1.70	34.23	89	Double Screened Stoker
13	00305	299/Black Creek	Raw	171	0.59	46.22	23	Double Screened Stoker
			Raw	465	0.53	34.95	15	Double Screened Stoker
			Raw	951	0.64	32.61	68	Double Screened Stoker
			Raw	979	0.73	46.33	78	Crushed Run-of-Mine
13	00750	100/Various	Raw	976	0.87	53.17	97	Crushed Run-of-Mine
13	07243	299/Black Creek	Washed	442	1.21	14.75	28	Double Screened Stoker
			Washed	897	1.15	17.96	61	Double Screened Stoker
16	02020	799/F	Raw	136	0.25	21.33	68	Double Screened Egg
			Raw	237	0.26	18.76	13	Single Screened Slack
			Raw	452	0.26	16.13	45	Single Screened Slack
			Raw	861	0.26	17.80	77	Single Screened Slack
20	07235	846/Hiawatha	Raw	140	0.48	17.98	67	Double Screened Stoker
			Raw	429	0.47	8.52	20	Double Screened Stoker
			Raw	76	0.53	11.37	126	Single Screened Slack

1/ Intervals specified for each mine were: 0-300 tons, 300-600 tons, and 600-1000 tons.

5.2.4 Mine 00614, Various Seams

Coal analyses were available for this mine on a raw and washed basis for the 600 to 1,000 ton interval. The average lbs S/MMBtu for the raw coal was calculated at 0.87 compared to 0.76 for the washed coal. However, the RSD's of the lbs S/MMBtu show no significant difference at 18.6 and 18.0 for the raw and washed coals, respectively.

5.2.5 Mine 00637, Upper Elkhorn #3 and Hazard #4 Seams

Analyses for this multiple seam operation were available for double screened and single screened washed coals. As shown in Table 24, there appears to be no significant difference between the average lbs S/MMBtu or the RSD of these two coals. In addition, no inverse relationship between RSD and lot-size is apparent.

5.2.6 Mine 02557, Upper Elkhorn #3 Seam

Data were available for both double screened and single screened washed coals. The double screened coals generally appear to have a lower average lbs S/MMBtu. On a lot-size basis, the data exhibit a direct relationship between lot-size and RSD.

Set out in Table 25 is a comparison of the washed coals from Mine 02557 to raw coals from the Upper Elkhorn seam. From Table 25 it appears that the raw coals have a lower average lbs S/MMBtu and a higher RSD than the washed coals.

Analyses for Mine 02557 were also available from data received from an electric utility. These analyses indicated

TABLE 25

COMPARISON OF DATA FROM MINE 02557 TO RAW COAL DATA FOR UPPER ELKHORN SEAM

Source	Preparation	Midpoint of Lot-Size	Lbs S/MMBtu Average	Lbs S/MMBtu RSD (%)	Number of Observations
Mine 02557	Washed, D.S.[1]	140	0.73	14.0	84
		472	0.72	19.7	84
		814	0.69	21.9	59
Mine 02557	Washed, S.S.[2]	93	0.82	11.9	53
		398	0.84	23.6	9
		902	0.78	18.8	113
Various Mines[3]	Raw	172	0.67	41.0	102
		443	0.59	27.5	96
		901	0.68	35.3	431

[1] Double screened.
[2] Single screened.
[3] From Table 21.

Source: Foster Associates, Inc.

an average of 0.64 lbs S/MMBtu with an RSD of 8.9 percent, based on shipments of approximately 2,600 tons each.

5.2.7 Mines 04184 and 04204, Hazard #5-A Seam

Mines 04184 and 04204 both produce coal from the Hazard #5-A seam. Mine 04184 is an auger operation producing a crushed run-of-mine product, while Mine 04204 is a strip mine producing double and single screened coals. In general there appears to be no significant difference in the average lbs S/MMBtu for these coals, although the RSD's for the raw coals appear to be slightly higher.

5.2.8 Mine 04978, Herrin #6 Seam

Analyses of this mine were based on raw and washed coals in the 600 to 1,000 ton lot-size. The raw coals exhibited an average of 1.8 lbs S/MMBtu with an RSD of 13.2 percent, while the washed coals had an average of 1.6 lbs S/MMBtu with an RSD of 9.32 percent.

Data for this mine received from an electric utility indicated an average of 2.8 lbs S/MMBtu with an RSD of 13.1 percent, based on washed coals with an average lot-size of approximately 2,000 tons.

5.2.9 Mine 07310, Morris #2 Seam

Bureau of Mines data for this mine indicated an average of 2.4 lbs S/MMBtu with an RSD of 10.5 percent, for the 600 to 1,000 ton lot-size. Similar data received from an electric utility provided an average of 2.5 lbs S/MMBtu with an RSD of 10.9 percent, based on an average lot-size of 4,700 tons.

5.2.10 Mine 04730, Indiana Seams #5, #6, and #7

Coal analyses for this multiple seam mine were based on washed, double screened stoker coals. By lot-size the average lbs S/MMBtu ranged from 1.0 to 1.7, while the RSD's ranged from 32.4 to 34.5 percent. Comparable data received from a coal producer indicated an average of 1.6 lbs S/MMBtu with an RSD of 34.3 percent, based on an average lot-size of 8,000 tons.

5.2.11 Mines 00305 and 07243, Black Creek Seam

Data for Mine 00305 were based on raw, double screened and crushed run-of-mine coals, while data for Mine 07243 were based on washed, double screened coals. The raw coals from Mine 00305 exhibited a lower average lbs S/MMBtu and higher RSD's compared to the washed coals from Mine 07243. The limited data for these mines did not permit an analysis of the relationship of RSD and lot-size.

5.2.12 Mine 00750, Various Seams

Based on crushed run-of-mine coals from a multiple seam operation, this mine exhibited an RSD of 53.2 percent for coal volumes ranging from 600 to 1,000 tons.

5.2.13 Mine 02020, F Seam

Analyses for this mine were based on raw, double and single screened coals. No significant difference was observed between the average lbs S/MMBtu for these coals, although the data indicated a slightly higher RSD for the double screened coal.

Comparable data for this mine received from an electric utility indicated an average of 0.54 lbs S/MMBtu, compared to 0.25 to 0.26 from the Bureau of Mines data. The RSD of the lbs S/MMBtu was calculated at 5.0 percent from the utility data, while the Bureau of Mines data indicated RSD's from 16.1 to 21.3 percent.

5.2.14 Mine 07235, Hiawatha Seam

Coals analyzed for Mine 07235 from the Bureau of Mines data were raw, double and single screened coals. By lot-size intervals the average lbs S/MMBtu ranged from 0.47 to 0.53 while the RSD's ranged from 8.5 to 18.0 percent. In comparison, data from an electric utility exhibited an average of 0.90 lbs S/MMBtu with an RSD of 27.6 percent, based on lot-size shipments of approximately 8,400 tons each.

5.2.15 Summmary of Analysis of Individual Mine Data from "Detail Tape"

In the analysis of the 16 selected mines from the U.S.B.M. "detail tape" the RSD's of lbs S/MMBtu generally ranged from 10 to 20 percent with two exceptions. First, Mine 04730 in Producing District 11 (Indiana) exhibited RSD's ranging from 32 to 34 percent. Second, Mines 00305 and 00750 in Producing District 13 (Alabama) exhibited RSD's ranging from 32 to 53 percent.

Based on the theoretical relationship between RSD and lot-size it was expected that rather large RSD's would be observed in the Bureau of Mines data, since all lot-sizes were 1,000 tons or less. As indicated above, relatively large RSD's were not observed and the results in general were similar to the results obtained from the respondents' data, which generally represented substantially larger lot-sizes.

In the 15 cases where two or more lot-size intervals were available for the same coal (same mine, same preparation, and same size) only three cases exhibited a consistent decline in RSD with lot-size.

The two mines for which raw and washed analyses were available indicated that the average lbs S/MMBtu as well as the RSD's were lower for washed coals.

In the seven cases where analyses were available for double screened and single screened coals from the same mine, there was no apparent consistent difference in the average lbs S/MMBtu or the RSD's.

Finally, the comparisons of Bureau of Mines data with data received from respondents for the same mines yield inconsistent results. Since the data from the respondents generally reflected larger lot-sizes, it was expected these data would exhibit lower RSD's than those observed in the Bureau of Mines data. For the seven mines compared, the respondents' data showed lower RSD's in two cases and higher RSD's in two cases. For the three remaining mines, the RSD's for the respondents' data were not significantly different from the Bureau of Mines data.

The results of these various comparisons must be viewed with caution, since the coal sampling and analyses were not performed by the same samplers or laboratories under controlled conditions. Moreover, the sporadic nature of the Bureau of Mines coal samples presents problems in statistical analysis. For this reason, it is questionable whether more sophisticated models of variance, including autocorrelative models, would be useful alternatives to the simple model used in this study.

6.0 Analysis of Continuous Monitoring Data for Sulfur Dioxide Emissions

The objective of this chapter was to analyze existing data for sulfur dioxide (SO_2) emissions and the removal efficiencies of flue gas desulfurization (FGD) systems. The analysis focused on the behavior of the variabilities of SO_2 emissions and FGD efficiencies as a function of averaging time. The behavior of the observed data was then compared to the expected behavior, based on statistical theory.

6.1 Description of Data

The data used in this study were collected by EPA and reflect the results of continuous monitoring test programs conducted at the Cane Run Unit No. 4, Bruce Mansfield Unit No. 1, Eddystone Unit No. 1, and Mitchell electric generating units. The data for the Mitchell unit were based on one-hour averages while the data for the three remaining units were based on 15-minute averages.

One advantage of using these data was that they were previously reduced, edited and reviewed. 1/ Since these data were collected under controlled conditions, it was possible to delete anomalous observations resulting from factors such as instrument and equipment malfunctions.

1/ A complete description of these data may be obtained from Air Pollution Emission Test, Volume I: First Interim Report: Continuous Sulfur Dioxide Monitoring at Steam Generators, U.S. EPA, Emissions Measurement Branch, EMB Report NO. 77SPP23A, August 1978.

6.2 Analysis of Data

The data analyses in this section were performed with a modified version of the analytical program developed for the analysis of coal data.

The results of the analysis of sulfur dioxide emissions and FGD efficiencies are set out in Table 26. As the table shows, the analysis consisted of calculating one-hour, three-hour, and twenty-four-hour simple averages as well as the corresponding moving averages. For the Mitchell and Cane Run Units the data also permitted the calculation of a 30-day moving average. The analysis is limited in its general applicability due to the limited number of data sets. However, even given the limited amount of data, some general observations can be made.

The first observation is that, while the FGD efficiencies are not subject to large relative variations, the relative variations exhibited in the FGD inefficiencies are substantial. This can be seen by comparing Columns (10) and (11) in Table 26. Based on a 24-hour averaging period, the RSD's of the FGD sulfur removal efficiencies ranged from 1.2 to 6.2 percent. In contrast, the RSD's of the FGD inefficiencies ranged from 10.5 to 70.8 percent for the same averaging period. Since emissions into the atmosphere are the product of inlet flue gas concentrations times FGD inefficiency, it appears that the relative variability in FGD performance is more important than the combined relative variabilities of the coal and combustion processes.1/

1/ It is also interesting to note that in a correlation analysis performed on the Cane Run Unit No. 4 data by the Energy Strategies Branch of EPA a correlation coefficient of 0.35 was obtained for inlet versus outlet emissions, based on three-hour averages. This correlation coeffient indicates that inlet and outlet emissions tend to vary independently about their respective means. This independence can be explained only by the variability in the performance of the FGD unit.

Table 26

ANALYSIS OF VARIABILITIES OF SULFUR DIOXIDE EMISSIONS AND FGD REMOVAL EFFICIENCIES AS A FUNCTION OF AVERAGING TIME PERIOD

Unit and Averaging Period (1)	Flue Gas Concentrations (Lbs SO_2/MMBtu)						FGD Efficiency (%)1/			FGD Inefficiency2/
	Inlet			Outlet						
	Mean (2)	S.D.3/ (3)	RSD (%) (4)	Mean (5)	S.D.3/ (6)	RSD (%) (7)	Mean (8)	S.D.3/ (9)	RSD (%) (10)	RSD (%) (11)
Eddystone No. 1										
1-Hour	5.1230	.4190	08.18	.2607	.2799	107.38	94.6551	5.4488	5.756	101.85
3-Hour	5.1232	.4100	08.00	.2597	.2578	99.28	94.6378	5.0120	5.295	93.47
3-Hour Moving	5.1213	.4091	07.99	.2584	.2583	99.97	94.6637	5.0259	5.309	94.18
24-Hour	5.1161	.3656	07.15	.2422	.1837	75.83	94.9278	3.5912	3.783	70.80
24-Hour Moving	5.1091	.3617	07.08	.2456	.1702	69.30	94.8830	3.3816	3.563	66.09
Mitchell										
1-Hour	6.5482	1.1886	18.15	.6737	.1832	27.20	89.9198	2.2469	2.498	22.29
3-Hour	6.7216	.9284	13.81	.6699	.1606	23.98	89.8779	1.8771	2.088	18.54
3-Hour Moving	6.7178	.9289	13.83	.6697	.1604	23.95	89.8785	1.8792	2.090	18.57
24-Hour	6.7150	.6988	10.41	.6697	.1089	16.26	89.8406	1.0779	1.199	10.61
24-Hour Moving	6.7194	.6827	10.16	.6702	.1094	16.33	89.8452	1.0696	1.190	10.53
30-Day Moving4/	6.7220	.0099	00.15	.6742	.0050	00.74	89.8043	0.0691	0.076	0.68
Mansfield No. 1										
1-Hour	6.6198	.7948	12.01	1.2699	.5433	42.79	80.8948	7.0020	8.655	36.81
3-Hour	6.6216	.7533	11.38	1.2692	.5137	40.47	80.9864	6.5540	8.092	34.48
3-Hour Moving	6.6212	.7566	11.43	1.2702	.5146	40.52	80.9704	6.5469	8.085	34.40
24-Hour	6.6187	.4486	06.78	1.2722	.3625	28.49	80.9522	4.7538	5.872	24.96
24-Hour Moving	6.6354	.4814	07.26	1.2867	.3606	28.03	80.7625	4.6223	5.723	24.03
Cane Run No. 4										
1-Hour	5.6435	.5412	09.59	.9100	.3669	40.32	83.6626	6.4201	7.673	39.30
3-Hour	5.6423	.5255	09.31	.9084	.3533	38.89	83.6663	6.1958	7.405	37.93
3-Hour Moving	5.6425	.5236	09.28	.9090	.3533	38.87	83.6582	6.2007	7.411	37.94
24-Hour	5.6398	.4203	07.45	.9059	.2892	31.92	83.6907	5.1752	6.183	31.73
24-Hour Moving	5.6433	.4108	07.28	.9096	.2859	31.43	83.6310	5.1775	6.190	31.63
30-Day Moving4/	5.6503	.0318	00.56	.9318	.0978	10.50	83.2279	1.7603	2.115	10.50

1/ FGD Efficiency = $$\frac{\text{Lbs } SO_2\text{/MMBtu Inlet} - \text{Lbs } SO_2\text{/MMBtu Outlet}}{\text{Lbs } SO_2\text{/MMBtu Inlet}}$$

2/ FGD Inefficiency = 1-Efficiency

3/ S.D. = Standard Deviation

4/ Analyses for 30-day moving average are based on very limited data.

Source: Foster Associates, Inc.

The RSD's of the inlet SO_2 concentrations ranged from 8.2 to 18.2 percent based on a one-hour averaging period. Although there are inherent difficulties in comparing coal analysis data to emissions data, as previously discussed, these RSD's are comparable to the RSD's of coal in the 0 to 300 ton range examined in the Bureau of Mines data.1/

A comparison of the FGD inlet data to outlet data shows that the outlet emissions are much more variable than the inlet emissions. Column (2) of Table 27 sets out the ratio of the RSD of inlet emissions to the RSD of outlet emissions for the various averaging periods examined for the four generating units. Based on the one-hour averaging periods, this ratio varies from 1.5 for the Mitchell unit to more than 13 for the Eddystone Unit No. 1. In other words, at the Eddystone unit, the relative variability of the outlet emissions is more than thirteen times greater than that of the inlet emissions. The implication of these results is that although the FGD unit reduces the mean emission rate, which aids compliance, the FGD unit also increases the relative variability of outlet emissions, which increases the difficulty of compliance.

A final observation is that the reduction in emissions variability resulting from increasing the averaging period is less than would be expected from statistical approximations, if independence is assumed. It can be shown that if moving or simple averages are calculated from one-hour averages with a standard deviation of σ, the standard deviation for a moving or a simple average of n data points would have a standard deviation of $\frac{\sigma}{\sqrt{n}}$. This relationship assumes that

---

1/ A 500 MW generating unit would consume coal at a rate of about 200 tons/hour.

TABLE 27

COMPARISON OF THE VARIABILITY OF FGD INLET AND OUTLET EMISSIONS, OBSERVED AND EXPECTED

	Sulfur Dioxide Emissions (Lbs SO_2/MMBtu)				
		Inlet		Outlet	
Unit and Averaging Period (1)	RSD Outlet / RSD Inlet (2)	Expected S.D.[1] (3)	Observed S.D. / Expected S.D. (4)	Expected S.D.[1] (5)	Observed S.D. / Expected S.D. (6)
Eddystone No. 1					
1-Hour	13.13	--	--	--	--
3-Hour	12.41	.2419	1.6949	.1616	1.5953
3-Hour Moving	12.51	.2419	1.6912	.1616	1.5984
24-Hour	10.61	.0855	4.2760	.0571	3.2172
24-Hour Moving	9.79	.0855	4.2304	.0571	2.9807
Mitchell					
1-Hour	1.50	--	--	--	--
3-Hour	1.74	.6862	1.3530	.1058	1.5180
3-Hour Moving	1.73	.6862	1.3537	.1058	1.5161
24-Hour	1.56	.2426	2.8805	.0374	2.9118
24-Hour Moving	1.61	.2426	2.8141	.0374	2.9251
30-Day Moving	4.93	.1276	.0776	.0199	.2513
Mansfield No. 1					
1-Hour	3.56	--	--	--	--
3-Hour	3.56	.4589	1.6415	.3137	1.6376
3-Hour Moving	3.55	.4589	1.6487	.3137	1.6404
24-Hour	4.20	.1622	2.7657	.1109	3.2687
24-Hour Moving	3.86	.1622	2.9679	.1109	3.2516
Cane Run No. 4					
1-Hour	4.20	--	--	--	--
3-Hour	4.18	.3125	1.6816	.2118	1.6681
3-Hour Moving	4.19	.3125	1.6755	.2118	1.6681
24-Hour	4.28	.1105	3.8036	.0749	3.8611
24-Hour Moving	4.32	.1105	3.7176	.0749	3.8171
30-Day Moving	18.75	.0767	.4146	.0528	1.8523

1/ S.D. = Standard Deviation

Source: Foster Associates, Inc.

the distribution of the one-hour averages is not serially correlated.[1]

Columns (4) and (6) of Table 27 compare the standard deviations of the observed moving and simple averages with the standard deviations which would be expected given the relationship and statistical assumption presented in the previous paragraph. From Table 27 it can be seen that in almost every case the expected standard deviation is less than the observed standard deviation. Thus, it appears that substantial reductions in emissions variability result from longer averaging periods, but these reductions are less than what would be expected under assumptions of statistical independence.

6.3 Implications of Emissions Analysis

To the extent that these four FGD units are representative of FGD units in general, the results of these analyses identify several factors which have an impact on compliance with sulfur dioxide emission regulations.

First, the reduction in emissions variability is readily apparent as the averaging interval is increased consecutively from one hour, to three hours, to 24 hours, to 30 days. These findings support the theoretical inverse relationship between coal sulfur variability and lot-size, since increasing the averaging interval is equivalent to increasing the increment or lot-size of coal burned.

---

[1] Yamane, Taro, Statistics, An Introductory Analysis, Harper and Row, Third Edition, p. 1072.

It follows from this analysis that, the shorter the averaging interval or the smaller the amount of coal burned per unit time, the more difficult it is to comply with an emission standard if the averaging basis is time. Moreover, the actual air pollution decreases as the difficulty of compliance increases. These findings are particularly relevant to small coal-fired plants and plants operating at coal-burn rates lower than the rates used to develop its compliance strategy. With respect to coal purchasing these plants would experience greater problems, especially the smaller plants with compliance strategies based on low-sulfur coal. While large sources might find it necessary to ensure that unit train loads (100 cars or approximately 10,000 tons) meet the standard, small sources would have to ensure compliance for perhaps two or three carloads. Due to the natural variability of coal, it is possible that, given the same source of coal supply, the coal would comply when burned in a large plant but would result in excess emissions in a small plant. Alternatively, the source of supply may be acceptable for the small plant, but the increased number of coal analyses and the selectivity required for quality control would certainly increase coal costs.

The second implication of the result of this analysis concerns those plants utilitizing FGD control strategies. Based on current technology, FGD is in most cases the only method available to meet stringent sulfur dioxide emission regulations. Although FGD units reduce the mean or average emission rate, it appears that they greatly increase the relative variability of the outlet emissions, which increases the difficulty of compliance. In the four units analyzed, it was found that the relative variability in emissions induced by FGD performance is significantly more important than the relative variabilities of the coal and the combustion process combined. As a consequence of these findings, the evaluation of a source of coal supply for a coal-fired plant equipped with an FGD unit must address not only the problem of natural coal sulfur variability, but also the variability in FGD performance. In general, this would require coal with a lower mean sulfur content than if it were assumed that the FGD unit operated at a constant rate of efficiency.

Finally, this analysis indicated that reductions in variability obtained from longer averaging intervals, although singificant, are less than would be expected based on assumptions of independence. These results suggested that an autocorrelative model may be more appropriate for outlet emissions. In addition, future studies of emissions analysis should address the effect of FGD units on extreme values as well as the relative variability.

7.0 Coal Sulfur Regression Analysis

7.1 Objectives

The purpose of the coal sulfur regression analysis is to examine the relationship between the sulfur content of coals and other coal characteristics. Because of the database limitations, these other coal characteristics are restricted to the following: ash content (AS), heat content (BT), moisture content (MO), lot-size (TON), mining method (MM), sampling method (SM), and the level of coal preparation (PC). The relationships between coal sulfur content and these other coal characteristics are examined for their statistical significance, consistency, and their explanatory or predictive power. Wherever data permit, the regression analysis is disaggregated to three levels: a Bureau of Mines Producing District, a seam in that producing district, and a mine in that seam. In addition to examining the relationship between the average sulfur content (SU) of coals and the other coal characteristics, the relationship between the coal sulfur variability, measured by the relative standard deviation or the variance (σ^2), and the other coal characteristics is examined.

7.2 Background

Sulfur does not occur as an element in coal, but as chemical combinations with other substances. Organic sulfur is combined with the organic coal substance and is part of the coal. Pyritic sulfur is combined with iron as either pyrite or marcasite. Sulfate sulfur is combined with either calcium or iron and is generally less than 0.05 percent of the coal. 1/

1/ F. E. Walker and F. E. Hartner, Forms of Sulfur in U.S. Coals, U.S. Bureau of Mines, IC 8301, 1966, p. 2.

Large differences in local and regional variability of sulfur frequently occur. Gomez, Donaven, and Hazen have performed a statistical and spatial evaluation of sulfur in coal seams and found very little correlation between the chemical and physical properties of the coal and sulfur concentration.[1] Figures 15 and 16 show the relationship between the average sulfur content and the various chemical and physical coal properties examined by Gomez, Donaven, and Hazen. No strong correlation between the sulfur content and ash content, moisture content, volatile matter, heat content, grindability, fixed carbon, free sweeling index, or ash softening temperature appears to exist. The results of their study led Gomez, Donaven, and Hazen to conclude, ". . . the chemical and physical properties of the coal are secondary variables influencing sulfur and mineral matter distribution in coal beds. The primary variables affecting sulfur distribution, quite likely, are geologic factors related to the depositional history of the seam."[2]

Discussions with Professor Joseph Leonard, Dean of the College of Mineral and Energy Resources at West Virginia University and Director of the Coal Research Bureau, and Dr. Francis Ting, professor of coal geology at West Virginia University, support the conclusions reached by Gomez, Donaven, and Hazen.[3] Both Professor Leonard and Dr. Ting agree that it is the geologic factors which determine the sulfur content of coals and not the chemical or physical properties of the coals themselves. Therefore, characteristics such as seam

---

[1] Manual Gomez, Donald J. Donaven, and Kathleen Hazen, The Statistical and Spatial Evaluation of Sulfur and Ash in Coal Seams, U.S. Bureau of Mines, RI 7679, 1972.

[2] Ibid., p. 3.

[3] Conversations with Professor Joseph Leonard and Dr. Francis Ting, West Virginia University, May 14, 1979.

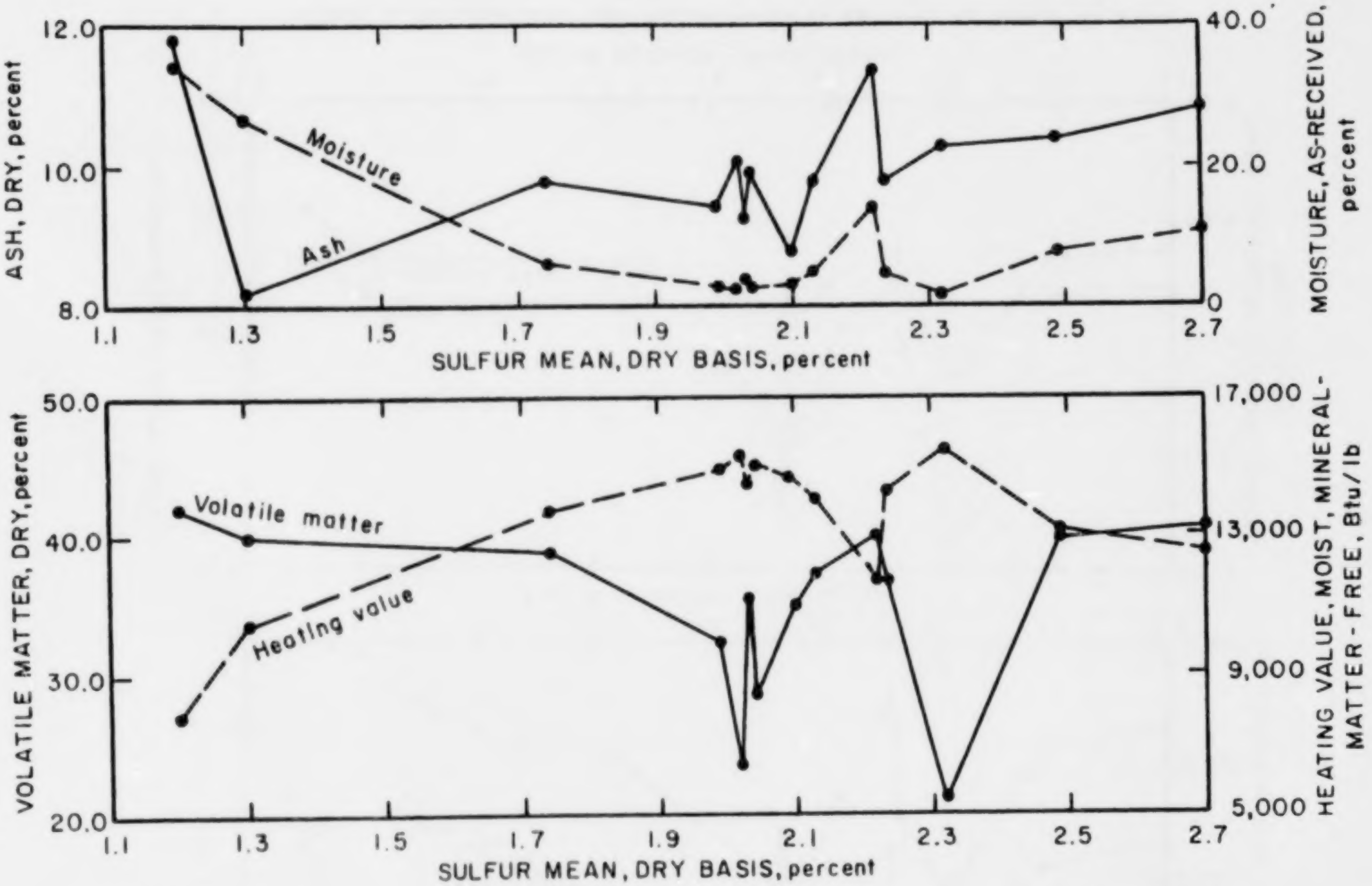


FIGURE 15 - Relationship Between Average Values for Volatile Matter, Ash, Moisture, Heating Value, and Sulfur.

Source: U.S. Bureau of Mines, RI 7679.

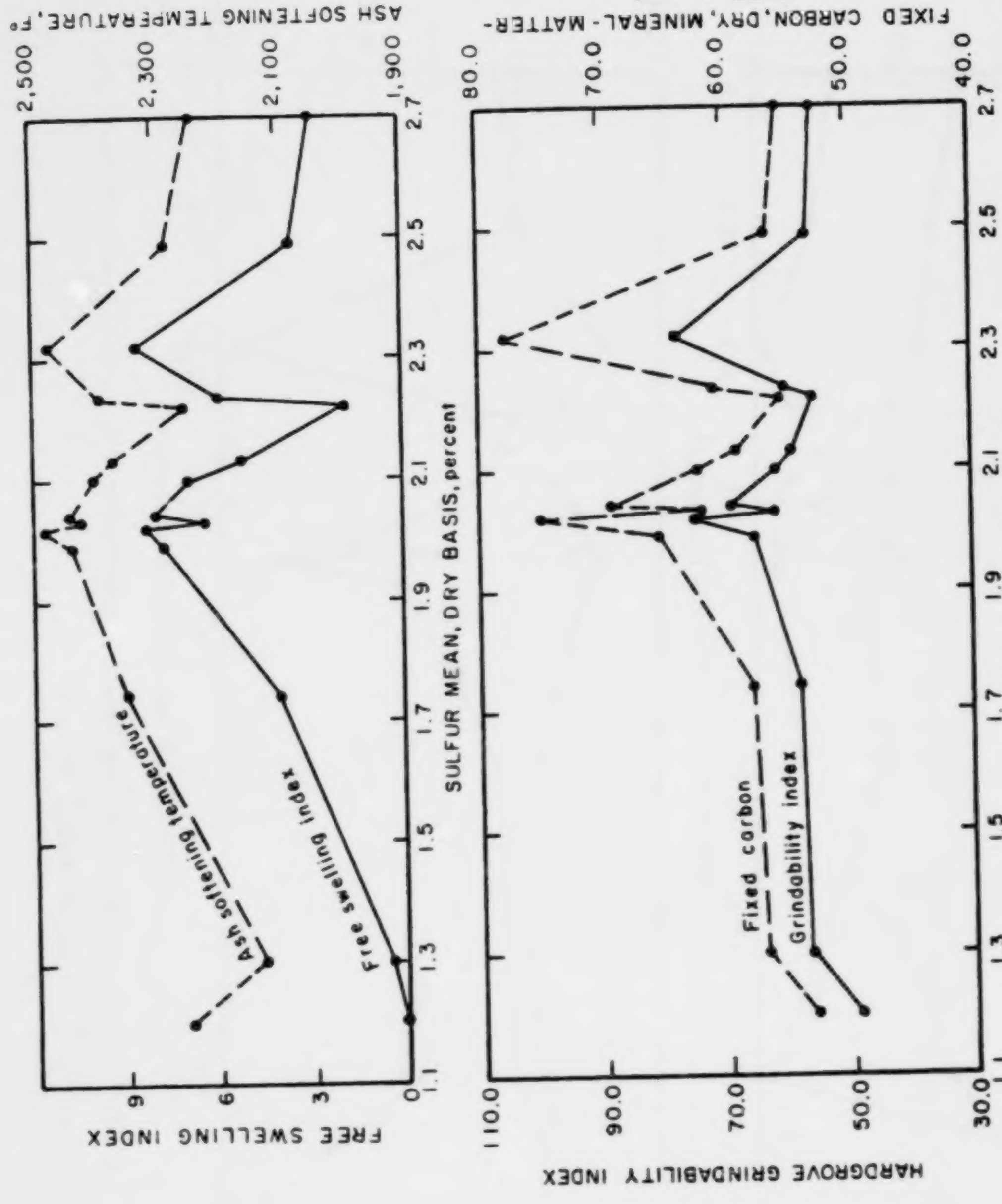


FIGURE 16 - Relationship Between Average Values for Ash Softening Temperature, Free-Swelling Index, Fixed Carbon, Hardgrove Grindability Index, and Sulfur.

Source: U.S. Bureau of Mines, RI 7679.

thickness, elevation, roof material, and type of deposition which are associated with geologic occurrences may have a stronger relationship to sulfur content than the chemical and physical properties of coal. For example, in the Eastern coal producing areas, lower sulfur coals are generally associated with thin seams, high elevation, clay roof, and freshwater deposition. Thin seams, high elevations, and freshwater are inferences of moving water environments which removed impurities from the coal. Clay roof materials are thought to have acted as a filter which prohibited impurities from penetrating the coalbed. Higher sulfur Eastern coals are generally associated with thick seams, low elevations, and marine deposition which are inferences of stagnant water environments where impurities were not removed from the coalbeds. However, the coal database developed for this study does not contain geologic factors and cannot be used to test the hypothesis of a greater correlation between sulfur content and geologic factors than between sulfur content and the chemical and physical properties of the coal.

7.3 Average Sulfur Content Regression Results

Linear regression analysis was used to examine the relationship between the average sulfur content of the coals and the other coal properties contained in the coal database. Where sufficient data were available to provide meaningful results, regressions were performed for individual coal producing districts, particular seams, and single mines. Coal Producing Districts 10 and 8 were disaggregated to the seam and mine levels, while Producing District 4 data did not permit analysis at the seam or mine levels. In Producing District 19 the regression analysis at mine level was based on core sample data and "as shipped" data.

7.3.1 Producing District 10

The coal analysis data for Producing District 10 were sufficient for regression analysis at the district level, the seam level, and the mine level. Regression equations for Producing District 10, the Illinois #6 Seam, and Mine #1 are shown below:

Producing District 10:

$$\mathrm{SU} = 40.11608 + \underset{(15.53)}{0.60056\ \mathrm{PC}} - \underset{(30.00)}{0.00270\ \mathrm{Btu}} - \underset{(1.48)}{0.00775\ \mathrm{SM}}$$

$$-\underset{(\text{undefined})^{*}}{0.00004\ \mathrm{TON}} + \underset{(14.18)}{0.21590\ \mathrm{MM}} - \underset{(28.03)}{0.37498\ \mathrm{MO}}$$

$$-\underset{(27.52)}{0.38478\ \mathrm{AS}}$$

$R^2 = 0.34258$ F = 335.58687 S.E. = 0.65832

Illinois #6 Seam:

$$\mathrm{SU} = 29.7889 + \underset{(44.83)}{1.04531\ \mathrm{MM}} - \underset{(22.16)}{0.47590\ \mathrm{PC}}$$

$$-\underset{(17.54)}{0.00194\ \mathrm{BT}} + \underset{(17.88)}{0.07689\ \mathrm{SM}} - \underset{(14.86)}{0.24858\ \mathrm{AS}}$$

$$-\underset{(12.52)}{0.22782\ \mathrm{MO}}$$

$R^2 = 0.55424$ F = 665.20574 S.E. = 0.58117

Mine #1:

$$\mathrm{SU} = 5.05320 - \underset{(6.00)}{0.00006\ \mathrm{TON}} + \underset{(3.08)}{0.05756\ \mathrm{AS}}$$

$$+ \underset{(7.58)}{0.60118\ \mathrm{PC}} - \underset{(3.30)}{0.06762\ \mathrm{SM}} - \underset{(2.85)}{0.05224\ \mathrm{MO}}$$

$$- \underset{(2.00)}{0.00024\ \mathrm{BT}} - \underset{(1.28)}{0.08957\ \mathrm{MM}}$$

$R^2 = 0.40888$ F = 69.17062 S.E. = 0.22051

*Note: T-statistics were estimated by dividing the regression coefficient of each variable by its corresponding standard error. By defintion the t-statistic was undefined when the standard error was zero.

where:
SU = sulfur content
AS = ash content
BT = heat content (Btu)
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code (Raw or Washed)

T-statistics for each regression coefficient are in parentheses. In most cases the t-statistics are significant at the 95 percent confidence level which indicates that the regression coefficients are statistically different from zero. Further information about each regression equation is presented in Table 28 which shows correlation coefficients, regression equation R^2, and variable rankings determined by a stepwise regression which orders each regression variable by its contribution to the explained variance of the equation.

Although the t-statistics of the regression coefficients and the F-statistics for the equations are generally significant at the 95 percent confidence level ($t = 1.96$, $F > 2.00$), the equations have very poor explanatory or predictive power as indicated by the low R^2 and high standard errors. Only about 34 percent of the total variance is accounted for by the regression equation for Producing District 10 as a whole. The regression equations account for about 55 percent and 41 percent of the total variance for the Illinois #6 Seam and Mine #1, respectively. High standard errors which range from about 0.66 to 0.22 also indicate weak explanatory or predictive powers for these equations. However, there is generally a trend toward higher R^2 and lower standard errors as one disaggregates from the Producing District level to the seam level to the mine level. These trends tend to support the

TABLE 28

SULFUR REGRESSION ANALYSIS SUMMARY FOR PRODUCING DISTRICT 10 (PRODUCING DISTRICT 10, ILLINOIS #6 SEAM, MINE #1)

	Correlation Coefficients		
Variable	Producing District 10	Illinois #6 Seam	Mine #1
	SU	SU	SU
SU	1.00000	1.00000	1.00000
AS	-0.01498	0.26644	0.17981
BT	-0.26196	-0.41158	-0.36402
MO	0.17751	0.11881	0.22398
TON	-0.16336	-0.24973	-0.46582
MM	0.20635	0.65276	0.44043
SM	-0.20061	0.14185	-0.31045
PC	0.31943	-0.06650	0.34396

	Regression Equation R^2		
Number of Variables	Producing District 10	Illinois #6 Seam	Mine #1
1	0.10203	0.42069	0.21699
2	0.16615	0.46088	0.32448
3	0.19855	0.50120	0.37546
4	0.21783	0.51899	0.39796
5	0.22517	0.53247	0.40462
6	0.23213	0.55424	0.40749
7	0.34258	0.55437	0.40888

	Regression Equation Variables		
Variable Rank	Producing District 10	Illinois #6 Seam	Mine #1
1	PC	MM	TON
2	BT	PC	AS
3	SM	BT	PC
4	TON	SM	SM
5	MM	AS	MO
6	MO	MO	BT
7	AS	TON	MM

SU = sulfur (as received)
AS = ash (as received)
BT = heat content (Btu)
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code (raw or washed)

a priori hypothesis that the fit of the regression equations should improve as the data are disaggregated and are more likely to be from similar populations at the seam and mine level than at the Producing District level. However, it should be noted that the highest R^2 occurs in the regression equation for the Illinois #6 Seam, while the lowest standard error occurs in the equation for Mine #1.

The regression equations and Table 28 indicate many inconsistencies in the values and signs of the correlation coefficients, the values and signs of the regression coefficients, and the order of the independent variables in the various levels of disaggregation. Only heat content (BT), moisture content (MO), lot-size (TON), and mining method (MM) have the same sign in each regression equation if the statistically insignificant values are ignored. At each level of disaggregation the sulfur content of the coal shows an inverse relationship to the heat content, moisture content, and lot-size and a positive relationship to the mining method. The moisture content parameter is somewhat of an oddity in that the moisture regression coefficient in each equation is negative, while the moisture correlation coefficient is positive. An explanation for this oddity may be that inclusion of other variables in the regression equations which exhibit stronger influences on the sulfur content than does the moisture content suppresses the positive relationship which the moisture content individually exhibits with sulfur content. Multicollinearity, which is the relationship between the explanatory variables, may also contribute to this oddity. No consistent relationship appears to exist at each level of disaggregation between the sulfur content and the remaining independent variables (ash, sampling method, preparation code).

Large differences also occur in the relative importance of each explanatory variable in each regression equation. Only one variable, preparation code (PC), appears in each equation as one of the three most important variables. However, each explanatory variable except moisture content (MO) appears at least once as one of the three most important variables in one of the equations. These differences in the relative importance of each explanatory variable in each regression equation is another indication that no strong and consistent relationship between sulfur content and the coal properties contained in the database exists for coals in Producing District 10.

7.3.2 Producing District 8

Sufficient data were available for Producing District 8 to disaggregate the data to a seam and a mine level. Regression equations for Producing District 8, the Alma (Blue Gem) Seam, and Mine #2 are shown below:

Producing District 8:

$$SU = -2.22768 - \underset{(32.14)}{0.15875\ PC} + \underset{(21.94)}{0.08315\ MM}$$

$$+ \underset{(\text{undefined})^{*}}{0.00007\ TON} + \underset{(7.87)}{0.05949\ AS} + \underset{(3.60)}{0.00018\ BT}$$

$$+ \underset{(1.49)}{0.01521\ MO}$$

$$R^2 = 0.43598 \qquad F = 260.62299 \qquad S.E. = 0.38109$$

*Note: T-statistics were estimated by dividing the regression coefficient of each variable by its corresponding standard error. By definition the t-statistic was undefined when the standard error was zero.

Alma Seam:
(Blue Gem)

$$SU = -2.58155 + \underset{(7.50)}{0.07662\ AS} + \underset{(4.80)}{0.00024\ BT}$$

$$R^2 = 0.16564 \quad F = 25.67525 \quad S.E. = 0.28988$$

Mine #2:

(1)
$$SU = 0.11525 - \underset{(6.16)}{0.09569\ MM} + \underset{(1.93)}{0.04916\ AS}$$

$$-\underset{(1.02)}{0.92574\ MO} + \underset{(2.00)}{0.00002\ TON} + \underset{(0.53)}{0.00008\ BT}$$

$$R^2 = 0.28895 \quad F = 27.06459 \quad S.E. = 0.24026$$

(2)
$$SU = 1.26368 - \underset{(6.33)}{0.09319\ MM} + \underset{(5.82)}{0.03656\ AS}$$

$$-\underset{(3.62)}{0.03751\ MO} + \underset{(2.00)}{0.00002\ TON}$$

$$R^2 = 0.28839 \quad F = 33.84014 \quad S.E. = 0.23999$$

where:
SU = sulfur content
AS = ash content
BT = heat content (Btu)
MO = moisture content
TON = lot-size
MM = mining method

The t-statistics for each regression coefficient are in parentheses. For the Producing District equation and the seam equation the t-statistics are generally significant at the 95 percent confidence level ($t > 1.96$) for each regression coefficient. However, only the mining method (MM) and the lot-size (TON) regression coefficients are significant in equation 1 for Mine #2. This is probably caused by the high collinearity primarily between the ash (AS) and heat content (BT) variables which have a correlation coefficient of -0.89155. Dropping the heat content variable (BT) from the equation does not significantly affect the value of R^2, F, or the standard error and results in significant regression coefficients for each variable (see mine #2, equation 2). Table 29 shows correlation coefficients, regression equation R^2, and variable rankings as determined by a forward stepwise regression.

At each level of disaggregation the regression equations show very poor explanatory or predictive power even though the regression coefficients and the equations themselves are significant as based on the t-tests and F-tests, respectively. Only about 44 percent of the total variance for Producing District 8 as a whole is accounted for by the regression equation. The explanatory or predictive power of the regression equations for the Alma (Blue Gem) Seam and for Mine #2 is even less, accounting for approximately 17 and 29 percent, respectively, of the total variance. Standard errors for the equations are fairly high ranging from about 0.43 to 0.29 and are further indications of the weakness of the explanatory or predictive power of these regression equations. Unlike the regression equations for Producing District 10, the goodness of fit of the regression equations for Producing District 8 do not tend to improve as one disaggregates the data to the seam and mine levels. The highest R^2 occurs at the Producing District level, although the standard errors decline with each level of disaggregation. Therefore, these regression results for Producing District 8 do not support the _a priori_ hypothesis that coal characteristics become more homogeneous and more easily predictable as the level of disaggregation increases.

Table 29 and the regression equations indicate many inconsistencies in the values and signs of the correlation coefficients and regression coefficients and in the order of

TABLE 29

SULFUR REGRESSION ANALYSIS SUMMARY FOR PRODUCING DISTRICT 8 (PRODUCING DISTRICT 8, ALMA (BLUE GEM) SEAM, MINE #2)

	Correlation Coefficients		
Variable	Producing District 8	Alma Seam	Mine #2
	SU	SU	SU
SU	1.00000	1.00000	1.00000
AS	-0.09804	0.33760	0.37465
BT	-0.00158	-0.18080	-0.27160
MO	-0.04998	-0.20457	-0.12180
TON	0.10708	0.18289	0.14291
MM	-0.04654	--	-0.43876
PC	-0.42106	--	--

	Regression Equation R^2		
Number of Variables	Producing District 8	Alma Seam	Mine #2
1	0.17729	0.11398	0.19251
2	0.28202	0.16518	0.24918
3	0.37631	0.16564	0.27194
4	0.42898	--	0.28839
5	0.43536	--	0.28895
6	0.43598	--	--

	Regression Equation Variables		
Variable Rank	Producing District 8	Alma Seam	Mine #2
1	PC	AS	MM
2	MM	BT	AS
3	TON	TON	MO
4	AS	--	TON
5	BT	--	BT
6	MO	--	--

SU = sulfur (as received)
AS = ash (as received)
BT = heat content (Btu)
MO = moisture content
TON = lot-size
MM = mining method
PC = preparation code

importance of the explanatory variables. The signs of the regression coefficients for ash (AS), heat content (BT), and lot-size (TON) are generally positive in the regression equations for Producing District 8, while they are negative in the Producing District 10 regression equations. Given the weak relationship which exists between sulfur content and these explanatory variables and the possible existence of multicollinearity, differences in the signs of the regression coefficients from one Producing District to another or from one equation to another within the same Producing District are not toally unexpected.

Inconsistencies also occur in the order of importance of each explanatory variable in these equations. No single variable appears in each equation as one of the three most important variables, as measured by each variable's contribution to the explained variance. However, every variable appears at least once as one of the three most important variables. The differences in the importance of each explanatory variable is another indication that the relationship between the sulfur content of coal and these other coal properties is weak and inconsistent and that these other coal properties are poor estimators of the sulfur content.

7.3.3 Producing District 4

For Producing District 4 the regression analysis was performed for the entire Producing District with no disaggregation to the seam or mine levels. The regression equation for Producing District 4 is shown below:

Producing District 4:

$$SU = 9.53445 - \underset{(23.62)}{0.13534}\ MO - \underset{(9.67)}{0.00029}\ BT$$

$$-\underset{(17.92)}{1.14779}\ PC + \underset{(8.00)}{0.00008}\ TON - \underset{(6.02)}{0.23392}\ MM$$

$$+ \underset{(2.93)}{0.00926}\ AS$$

$R^2 = 0.24987$ F = 352.15370 S.E. = 0.76893

where:
SU = sulfur content
AS = ash content
BT = heat content (Btu)
MO = moisture content
TON = lot-size
MM = mining method
PC = preparation code (raw or washed)

T-statistic values for each regression coefficient are in parentheses. Each regression coefficient is statistically significant at the 95 percent confidence level. The regression equation R^2 is low, indicating that the regression equation accounts for only about 25 percent of the total variance, while the standard error is quite large at approximately 0.77.

Variable rankings and the respective regression equation R^2 for Producing District 4 are shown in the following table.

TABLE 30

Variable Order	Explanatory Variable	Regression Equation R^2
1	MO	0.10316
2	BT	0.19615
3	PC	0.23768
4	TON	0.24934
5	MM	0.24886
6	AS	0.24987

The three most important variables, moisture content (MO), heat content (BT), and the preparation code (PC) have never appeared together as the three most important variables in any regression equation thus far. Heat content and preparation code variables have frequently been one of the three most important explanatory variables in other equations. However, the moisture content variable appeared as one of the three most important variables in only one other equation -- Mine #2 in Producing District 8.

7.3.4 Mine #3

The data for Mine #3 in Campbell County, Wyoming consists of core sample analyses and "as shipped" analyses with the core analyses generally having a higher average sulfur content than the "as shipped" analyses. Regression equations for the core sample data and the "as shipped" data are shown below:

Mine #3:
(Core data)

$$SU = -0.88979 - \underset{(14.23)}{0.02832\ MO} + \underset{(12.00)}{0.00024\ BT}$$

$$+ \underset{(8.60)}{0.02889\ AS}$$

$$R^2 = 0.23213 \qquad F = 154.47654 \qquad S.E. = 0.06152$$

Mine #3:
("as shipped" data)

$$SU = -1.82902 + \underset{(24.00)}{0.00029\ BT} + \underset{(8.17)}{0.02451\ AS} - \underset{(\text{undefined})^*}{0.0000\ TON}$$

$$+\underset{(3.00)}{0.00162\ MO}$$

*Note: T-statistics were estimated by dividing the regression coefficient of each variable by its corresponding standard error. By definition the t-statistic is undefined when the standard error is zero.

$R^2 = 0.24992$ $F = 152.85478$ S.E. $= 0.04323$

where:
SU = sulfur content
AS = ash content
BT = heat content
MO = moisture content
TON = lot-size

The value of the t-statistic for each regression coeffficent is in parentheses. Each regression coefficient is significant at the 95 percent confidence level. Sulfur correlation coefficients, regression equation R^2, and variable rankings by order of importance are shown in Table 31.

Similar to the other regression equations, the Mine #3 regression equations have low R^2 values, indicating very weak explanatory or predictive power. The regression equation based on "as shipped" analyses does have slightly more explanatory power than the regression based on core analyses. Although the Mine #3 regression equations have very weak explanatory power, the low R^2 values are not accompanied by the high standard errors which have generally occurred in the other regression equations. The standard errors in the Mine #3 equations are the lowest for any equation. Another interesting inconsistency is the ranking of the moisture content variable (MO) in each equation. Moisture is the most important explanatory variable in the core sample regression equation and the least important in the "as shipped" sample regression equation. The sign of the moisture regression coefficient is different in each equation, being negative in the core sample equation and positive in the "as shipped" sample equation. The existence of multicollinearity between the explanatory variables may be at least partially responsible for these inconsistencies.

TABLE 31

SULFUR REGRESSION ANALYSIS SUMMARY FOR MINE #3 (MINE #3, CAMPBELL COUNTY, WYOMING)

	Sulfur Correlation Coefficients	
Variable	Core Samples	"As Shipped" Samples
	SU	SU
SU	1.00000	1.00000
AS	-0.13969	-0.12402
BT	0.30040	0.45812
MO	-0.39697	-0.07688
TON	0.00000	-0.17122
MM	--	--
PC	--	--

	Regression Equation R^2	
Number of Variables	Core Samples	"As Shipped" Samples
1	0.15769	0.20987
2	0.19516	0.23202
3	0.23213	0.24524
4	--	0.24992

	Regression Equation Variables	
Variable Rank	Core Samples	"As Shipped" Samples
1	MO	BT
2	BT	AS
3	AS	TON
4	--	MO

SU = sulfur content
AS = ash content
BT = heat content (Btu)
MO = moisture content
TON = lot-size
MM = mining method
PC = preparation code

7.3.5 Summary of Average Sulfur Content Regression Results

Tables 32 and 33 show the sign of each regression coefficient and the order of importance of each explanatory variable for each regression equation. In general there is very little consistency in the sign of the regression coefficients either between each coal Producing District or within each coal Producing District. No explanatory variable has the same sign in each equation. The regression coefficients for the ash content variables (AS) and the moisture content variables (MO) show the most consistency. In seven of the nine equations the regression coefficient of the ash content is positive while the regression coefficient of the moisture content is negative in six of the eight equations in which the moisture variable appears. The other variable regression coefficients are generally about equally divided between positive and negative values. Similar inconsistencies also occur in the order of importance of each variable. No single variable appears among the three most important variables in each quation. The variable that appears most often among the three most important explanatory variables is heat content (BT), but this variable appears only six times.

Considering these inconsistencies in the sign of the regression coefficients and in the rank of the explanatory variables along with the generally low R^2 values and high standard errors leads to the conclusion that the sulfur content of coals cannot be explained or predicted with any degree of confidence by regression equations in which the physical and chemical properties of coal are the explanatory variables. Based on these regression results, the sulfur content appears to be largely uncorrelated with the other physical and chemical coal properties. If the sulfur

TABLE 32

SIGN OF THE REGRESSION COEFFICIENTS

	Producing District 10			Producing District 8				Mine #3	
Explanatory Variable	Dist. 10	Illinois #6 Seam	Mine #1	Dist. 8	Alma (Blue Gem) Seam	Mine #2	Dist. 4	Core Analyses	"As Shipped" Analyses
AS	-	-	+	+	+	+	+	+	+
BT	-	-	-	+	+	+	-	+	+
MO	-	-	-	+	NA	-	-	-	+
TON	-	-	-	+	NA	+	+	NA	NA
MM	+	+	NA	+	NA	-	-	NA	NA
SM	-	+	-	NA	NA	NA	NA	NA	NA
PC	+	-	+	-	NA	NA	-	NA	NA

NA = not applicable.

AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code

TABLE 33

EXPLANATORY VARIABLE

	Producing District 10			Producing District 8				Mine #3	
Variable Rank	Dist. 10	Illinois #6 Seam	Mine #1	Dist. 8	Alma (Blue Gem) Seam	Mine #2	Dist. 4	Core Analyses	"As Shipped" Analyses
1	PC	MM	TON	PC	AS	MM	MO	MO	BT
2	BT	PC	AS	MM	BT	AS	BT	BT	AS
3	SM	BT	PC	TON	TON	MO	PC	AS	TON
4	TON	SM	SM	AS	--	TON	TON	--	MO
5	MM	AS	MO	BT	--	BT	MM	--	--
6	MO	MO	BT	MO	--	--	AS	--	--
7	AS	TON	MM	--	--	--	--	--	--

AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code

content can in fact be explained or predicted from other variables, these other variables are not the coal characteristics generally determined from coal analyses. These other variables may be geologic variables as hypothesized by Gomez, Donaven, and Hazen and others. However, coal analyses which contain geologic data are very limited at this time and are not contained in the coal database assembled for this study.

7.4 Sulfur Variability Regression Results

Since the primary objective of this study was to examine the variability of sulfur in coal, regression analysis was used to examine the relationship between the sulfur variability and the other physical and chemical coal properties contained in the coal database. Two measures of sulfur variability are used in this regression analysis. One is the relative standard deviation of sulfur and the other is the variance (σ^2) of sulfur. All regressions were performed for the Illinois #6 seam.

7.4.1 Relative Standard Deviation Regressions

Two different regression analyses were examined for the RSD of sulfur. First, the RSD was considered to be a linear function of all coal properties except sulfur content (SU). In the second regression analysis sulfur content was included as an explanatory variable. The regression equations are shown below:

Illinois #6 Seam:
(excluding sulfur)

$$\text{RSD} = -37.73231 + \underset{(2.80)}{5.65541\ \text{SM}} + \underset{(2.18)}{3.05614\ \text{AS}}$$

$$- \underset{(1.16)}{0.00100\ \text{TON}} + \underset{(0.98)}{6.43196\ \text{MM}}$$

$$R^2 = 0.60573 \quad F = 3.84077 \quad \text{S.E.} = 8.92782$$

Illinois #6 Seam:
(including sulfur)

$$\begin{aligned} \mathrm{RSD} = {} & -54.92410 - \underset{(4.54)}{13.88940\ \mathrm{SU}} + \underset{(3.62)}{4.63748\ \mathrm{AS}} \\ & \underset{(1.03)}{-0.00106\ \mathrm{TON}} - \underset{(0.56)}{0.00037\ \mathrm{BT}} + \underset{(2.64)}{3.92511\ \mathrm{MO}} \\ & + \underset{(2.40)}{5.87215\ \mathrm{SM}} + \underset{(1.19)}{5.40324\ \mathrm{MM}} - \underset{(0.51)}{2.85861\ \mathrm{PC}} \end{aligned}$$

$$R^2 = 0.91745 \qquad F = 8.33555 \qquad \mathrm{S.E.} = 5.27383$$

where:
RSD = relative standard deviation of sulfur
AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code

Regression coefficient t-statistics are in parentheses. When the sulfur content was excluded, the regression coefficients for the lot-size (TON) and mining method (MM) variables were insignificant at the 95 percent confidence level. From Table 34 it can be seen that the regression coefficients of the explanatory variables become insignificant as more explanatory variables are included. Multicollinearity between the independent variables may be the cause of the change in the significance of the regression coefficients as more explanatory variables are added. The addition of explanatory variables does not have a significant effect on the regression equation R^2 or standard errors. If the only explanatory variables considered are sampling method (SM) and ash content (AS), the equation R^2 is about 0.52 and the standard error is about 8.95 compared to an R^2 of approximately 0.63 and a standard error of 10.28 when all variables are included.

TABLE 34

REGRESSION RESULTS SUMMARY FOR THE ILLINOIS #6 SEAM

Regression Analysis Excluding Sulfur Content as an Explanatory Variable

Variable Order	Explanatory Variable	Regression Equation R^2	Standard Errors
1	SM	0.30198	10.41861
2	AS	0.52488	8.94659
3	TON	0.56806	8.90969
4	MM	0.60573	8.92782
5	MO	0.62701	9.15319
6	BT	0.63344	9.62436
7	PC	0.63399	10.28122

Regression Analysis Including Sulfur Content as an Explanatory Variable

Variable Order	Explanatory Variable	Regression Equation R^2	Standard Errors
1	SU	0.48643	8.93667
2	AS	0.68482	7.28679
3	TON	0.76348	6.59301
4	BT	0.79831	6.38549
5	MO	0.81879	6.37995
6	SM	0.89794	5.07845
7	MM	0.91392	4.98589
8	PC	0.91745	5.27383

SU = sulfur content
AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code

The addition of sulfur content as an explanatory variable improves the regression equation R^2 almost 50 percent to nearly 0.92 and reduces the standard error by about the same percent to 5.28. Such an improvement in the regression equation is expected since a fairly strong relationship should exist between the RSD of sulfur and the sulfur content because the average sulfur content is a component of the RSD ($RSD = \sigma/\bar{x}$). However, the inclusion of the sulfur content variable presents problems of a statistical nature. First, the existence of multicollinearity may be increased because the previous regression analyses showed that sulfur content has some relationship, although weak, to the other physical and chemical coal properties. As a consequence of this multicollinearity the t-statistics and the signs of the regression coefficients may be unreliable, and the regression coefficients may be highly sensitive to the particular sets of data and the number of observations. Inferences about the degree or seriousness of the multicollinearity can be obtained by using the Farrar-Glauber test for multicollinearity.1/

Another problem created by the inclusion of sulfur content as an explanatory variable is the possible existence of heteroskedasticity. Heteroskedasticity exists when the variance of the disturbance term is not constant. Since the RSD of sulfur is inversely related to the average sulfur content, the variance of the disturbance term may not be constant for all observations. Heteroskedasticity results in the least squares estimators not having the smallest variance and, therefore, provides least squares estimators of the regression coefficients which are inefficient -- not the best linear unbiased estimates. One method of testing

---

1/ Johnston, Econometric Methods, 2nd Ed., McGraw-Hill Book Company, 1972, pp. 163-164.

for the existence of heteroskedasticity is a nonparametric test proposed by Goldfield and Quant.[1] Although the possible presence of multicollinearity and heteroskedasticity make the actual contribution of each explanatory variable unreliable, the high R^2 value indicates that the regression equation containing sulfur content as an explanatory variable has strong predictive powers accounting for almost 92 percent of the total variance. However, with a standard error of about 5.3 the predicted RSD would be in the range of RSD ± 10.3 at the 95 percent confidence level. This is a large range since the mean RSD for the Illinois #6 seam is about 13.7 percent.[2]

7.4.2 Variance (σ^2) Regressions

In an attempt to reduce the possible existence of heteroskedasticity a regression analysis was performed using the variance of sulfur as the dependent variable. The regression equation for the Illinois #6 Seam is shown below:

Illinois #6 Seam:

$$\begin{aligned}\sigma^2 = -0.89731 &+ \underset{(3.21)}{0.09950}\ \mathrm{SM} + \underset{3.77)}{0.06120}\ \mathrm{AS} \\ &- \underset{(1.88)}{0.07317}\ \mathrm{SU} + \underset{(2.16)}{0.04069}\ \mathrm{MO} - \underset{(2.00)}{0.00002}\ \mathrm{TON} \\ &- \underset{(1.00)}{0.00001}\ \mathrm{BT} - \underset{(0.86)}{0.0160}\ \mathrm{PC} + \underset{(0.78)}{0.04493}\ \mathrm{MM}\end{aligned}$$

---

[1] Johnston, Econometric Methods, 2nd Ed., McGraw-Hill Book Company, 1972, pp. 218-219.

[2] For example, based on a 1.2 lbs SO_2/MMBtu standard, Table 10 in Chapter 2 shows that the mean level of emissions required to meet the standard would range from 1.06 to 0.73 lbs SO_2/MMBtu for respective RSD's of 5 and 25 percent, assuming a normal distribution.

$R^2 = 0.08834$ $F = 5.68211$ $S.E. = 0.06696$

where: σ^2 = variance of sulfur
SU = sulfur content
AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code

Values of the t-statistic for each regression coefficient are in parentheses. Many of the regression coefficients are insignificant at the 95 percent confidence level with only the sampling method (SM), ash content (AS), and moisture content (MO) variables having significant t-statistics. However, because of the possible existence of multicollinearity, especially with the inclusion of sulfur content (SU) as an explanatory variable, the regression coefficients are suspect as to their values and significance. Removal of the insignificant variables, as indicated in the following table, does not cause major changes in the R^2 values or the standard errors. The R^2 is about 0.73 and the standard error about 0.072 when only sampling method (SM) and ash content (AS)

TABLE 35

Variable Order	Explanatory Variable	Regression Equation R^2	Standard Error
1	SM	0.31056	0.11062
2	AS	0.73325	0.07162
3	SU	0.74853	0.07263
4	MO	0.83410	0.06187
5	TON	0.85464	0.06104
6	BT	0.86562	0.06225
7	PC	0.87161	0.06505
8	MM	0.88340	0.06696

$R^2 = 0.08834 \quad F = 5.68211 \quad S.E. = 0.06696$

where: σ^2 = variance of sulfur
SU = sulfur content
AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = preparation code

Values of the t-statistic for each regression coefficient are in parentheses. Many of the regression coefficients are insignificant at the 95 percent confidence level with only the sampling method (SM), ash content (AS), and moisture content (MO) variables having significant t-statistics. However, because of the possible existence of multicollinearity, especially with the inclusion of sulfur content (SU) as an explanatory variable, the regression coefficients are suspect as to their values and significance. Removal of the insignificant variables, as indicated in the following table, does not cause major changes in the R^2 values or the standard errors. The R^2 is about 0.73 and the standard error about 0.072 when only sampling method (SM) and ash content (AS)

TABLE 35

Variable Order	Explanatory Variable	Regression Equation R^2	Standard Error
1	SM	0.31056	0.11062
2	AS	0.73325	0.07162
3	SU	0.74853	0.07263
4	MO	0.83410	0.06187
5	TON	0.85464	0.06104
6	BT	0.86562	0.06225
7	PC	0.87161	0.06505
8	MM	0.88340	0.06696

are the explanatory variables, compared to an R^2 of 0.88 and a standard error of 0.067 when all variables are included. Although the contribution of each explanatory variable is suspect, the high R^2 values indicate that this regression equation has good predictive power because it accounts for over 88 percent of the total variance. However, given a standard error of about 0.067, the predicted variance would be between $\sigma^2 \pm 0.13$ at the 95 percent confidence level. This is a very large range considering that the mean variance in the coal samples for the Illinois #6 Seam is approximately 0.13.

7.4.3 Summary of Sulfur Variability Regression Results

Table 36 shows the sign of each regression coefficient and the order of importance of each explanatory variable. Much more consistency appears in the signs of the regression coefficients in these equations than in the equations where sulfur content was the dependent variable. The signs of the regression coefficients are the same in each equation for six of the eight explanatory variables. Only the heat content (BT) and preparation code (PC) regression coefficients do not have the same sign in each equation. More consistency also occurs in the relative importance of the explanatory variables. Ash content (AS) is the second most important variable in each equation. Three other variables -- sampling method (SM), lot-size (TON), and sulfur content (SU) -- appear as one of the three most important variables in two of the three equations. The frequent occurrence of the lot-size variable among the three most important explanatory variables and the negative sign of the lot-size regression coefficient lends support to the hypothesis that the relative standard deviation of sulfur tends to decrease as the lot-size increases.

TABLE 36

REGRESSION EQUATION SUMMARIES FOR SULFUR VARIANCE IN ILLINOIS #6 SEAM

	Sign of the Regression Coefficients		
	Relative Standard Deviation (RSD) Equations		
Explanatory Variable	Excluding Sulfur Content	Including Sulfur Content	Variance (σ^2) Equation
SU	NA	-	-
AS	+	+	+
BT	+	-	-
MO	+	+	+
TON	-	-	-
MM	+	+	+
SM	+	+	+
PC	-	-	-

	Explanatory Variables		
	Relative Standard Deviation (RSD) Equations		
Variable Rank	Excluding Sulfur Content	Including Sulfur Content	Variance (σ^2) Equation
1	SM	SU	SM
2	AS	AS	AS
3	TON	TON	SU
4	MM	BT	MO
5	MO	MO	TON
6	BT	SM	BT
7	PC	MM	PC
8	--	PC	MM

Note: The statistical significance of the variables is not considered.

NA = not applicable.

SU = sulfur content
AS = ash content
BT = heat content
MO = moisture content
TON = lot-size
MM = mining method
SM = sampling method
PC = prepartion code

Although the problems of multicollinearity and heteroskedasticity are more likely to occur in the regression analysis of the relative standard deviation of sulfur and the variance of sulfur, the R^2 values are much higher and the standard errors are much lower in the RSD and σ^2 regression equations than in the sulfur content (SU) regression equations. The RSD and σ^2 regression equations account for approximately 90 percent of the total variance when all explanatory variables are included and account for about 70 percent of the total variance when only the two most important explanatory variables are included. Although the presence of multicollinearity and heteroskedasticity make the individual regression coefficients unreliable and suspect for explanatory purposes, the equations as a whole have high predictive power as measured by the equation R^2. However, the large ranges in which the predicted RSD and σ^2 would fall at the 95 percent confidence level greatly reduce the usefulness at these equations for predicting either the RSD or the σ^2 of sulfur.

8.0 Conclusions and Recommendations

8.1 Conclusions

1. The coal data collected for this report was analyzed by sorting by lot-size intervals and comparing the RSD of lbs S/MMBtu versus lot-size. The data were analyzed by U.S. Bureau of Mines Producing District, coal seam, and individual mine on a raw and washed basis. For individual mines, which generally had data available for only a few lot-size intervals, the results exhibited no consistent relationship between RSD and lot-size. As the data were aggregated to seam and Producing District, the results were still inconsistent, but in the majority of the cases the results exhibited an increase in the RSD of lbs S/MMBtu for successively smaller lot-sizes. These results provide limited support for an inverse relationship between RSD and lot-size.

2. Various regression analyses of the coal data provided limited support for an inverse relationship between the RSD of lbs S/MMBtu and lot-size.

3. The results of a simulation model, which was developed to examine coal sulfur variability, indicated that theoretically, coal sulfur variability should decrease with increasing lot-sizes.

4. An analysis of stack monitoring data from four electric generating units indicated that significant reductions in the relative variability of sulfur dioxide emissions can be achieved by using longer averaging intervals. It follows from this analysis that the smaller the

amount of coal burned per unit time, the more difficult it is to comply with an emission standard if the averaging basis is time. In addition, these results support the existence of an inverse relationship between the RSD of lbs S/MMBtu and lot-size, since increasing the averaging interval is equivalent to increasing the volume or lot-size of coal burned.

5. Sulfur dioxide emission regulations that require a probability of a very low number of days of excess emissions per year (for example, one or two days per year), require extremely high probabilities of compliance on the individual days of the year and substantially reduce the average level of emissions required for compliance.

6. Although flue gas desulfurization (FGD) decreases the mean level of sulfur emissions, the limited data analyzed showed that the relative variability of the emissions increases. In one case examined the relative variability of outlet emissions was more than thirteen times greater than the inlet emissions.

7. Composite coal seam or Producing District data cannot be used to accurately predict the variability of lbs S/MMBtu for individual mines within the coal seams or Producing Districts. Both seam and Producing District data provide biased estimates which consistently overestimate the RSD of lbs S/MMBtu for individual mines. Even if a scaling factor were used, the composite estimates would not reasonably predict mine variabilities.

8. The overall frequency distributions of lbs S/MMBtu and coal sulfur contents (weight percent) are skewed to the

right and are best represented by the inverted gamma distribution which appeared to be slightly superior to the lognormal distribution, and definitely superior to the normal distribution.

9. In the extreme right tail of the frequency distribution for lbs S/MMBtu (top 1.5 percent of the distribution) the data provided ambiguous results with respect to the best choice between the lognormal and the inverted gamma distributions.

10. The overall frequency distributions of coal heat contents (Btu/lb) appeared to be reasonably symmetrical and are closely approximated by the normal distribution.

11. The normal, lognormal, and inverted gamma distributions provided similar estimates for the mean lbs SO_2/MMBtu required for compliance with stringent sulfur dioxide emmission regulations. However, these distributions provided significantly different estimates under the assumptions of less stringent emission limits and coals with large RSD's for lbs S/MMBtu.

12. Comparisons of raw and washed coals on a mine, seam, and Producing District basis consistently indicated lower average lbs S/MMBtu as well as lower RSD's for the washed coals.

13. Based on the data analyzed, the type of frequency distributions for washed coal characteristics are not significantly different from the type of frequency distributions for raw coal characteristics.

14. Within individual mines, no significant differences were observed in the lbs S/MMBtu and RSD's for double screened and single screened coals.

15. Measurement errors in ASTM sampling and analysis procedures resulted in biased estimates which consistently overestimated the true RSD of lbs S/MMBtu. The differences between the measured and true RSD's are most significant for low measured RSD's. Theoretical calculations indicated that at a measured RSD of 7 to 8 percent, the true RSD was approximately zero.

16. Coal sulfur variability is a result of many interrelated factors. Statistical analysis of several of the factors believed to contribute to sulfur variability failed to identify any consistent, predictable relationship.

17. Various regression analyses based on mines, seams, and Producing Districts indicated that neither coal sulfur contents (weight percent) nor coal sulfur variabilities can be accurately predicted from the database developed in this study. The results tend to support the hypothesis that the primary factors affecting coal sulfur distributions are geologic factors related to the depositional history of the coal, while chemical and physical properties of coal are secondary factors influencing coal sulfur distributions.

18. Discussions with coal companies, Federal agencies, and research organizations did not reveal the existence of data which would permit an examination of the relationship between coal sulfur variability and geological factors or mining techniques. Further, these discussions did not reveal the existence of reliable data which

would permit an accurate assessment of the correlation between stack emissions and coal analyses, or between raw and washed coals.

19. The various analyses performed in this study identified no reliable method for coal suppliers, coal consumers, or air pollution control agencies to predict coal sulfur variability, which is often critical for compliance with existing sulfur dioxide regulations. Coal sulfur variability is especially critical for small coal-fired boilers subject to regulations that specify short averaging time intervals. The findings of this report suggested that the requirements of many current sulfur dioxide regulations are not consistent with the state of knowledge concerning coal sulfur variability.

8.2 Recommendations

1. Additional studies should be performed using more sophisticated models, such as autocorrelative models, which may yield more useful results than classical statistical models assuming independence.

2. This study investigated the goodness of fit between the observed distributions of coal characteristics and the normal, lognormal, and inverted gamma distributions. Additional studies should be performed to examine the goodness of fit for other skewed distributions, especially in the extreme right tail which becomes increasingly important when a high probability of not exceeding an upper limit is required.

3. Geostatistical methods, which would take into account both structure and randomness, should be used to

investigate the individual processes which influenced the coal from the time it was deposited to the time it was burned. These processes include depositional environment, in situ variability, mining methods, blending, cleaning, burning, and desulfurization.

4. Controlled experiments, although a major undertaking, might be performed to obtain the quality of data required to investigate the various processes which influence coal sulfur and sulfur dioxide emissions variability. The limited data currently available are observational data used for establishing coal prices and monitoring overall coal quality.

5. A comprehensive model would be useful to assess the impact on air quality. Inputs to this model would include parameters for coal characteristics, mining and handling methods, combustion and control equipment, meteorological data, and other variables.

6. Alternative sulfur dioxide emission regulations, which would mitigate the impact of coal sulfur variability yet achieve the objectives of existing regulations, should be investigated.

APPENDIX A

DERIVATION OF EQUATIONS USED FOR THE DEVELOPMENT OF TABLES 10, 11, AND 12

This technical appendix sets out the formulas and approximations used to derive Tables 10, 11, and 12 in this study.

TABLE 10, NORMAL DISTRIBUTION

(1) $$Z = \frac{S_{max} - \mu_x}{\sigma_x}$$

where:
S_{max} = Emission standard, lbs SO_2/MMBtu
μ_x = Required mean, lbs SO_2/MMBtu
σ_x = Standard deviation

(2) $$RSD = \frac{\sigma_x}{\mu_x}$$

Let $Z = 2.57583$, which corresponds to a 0.005 probability of values higher than S_{max}.

(3) Therefore: $$2.57583 = \frac{S_{max} - \mu_x}{(RSD)\ (\mu_x)}$$

(4) And: $$\mu_x = \frac{S_{max}}{1 + 2.57583\ (RSD)}$$

TABLE 11, LOGNORMAL DISTRIBUTION

From Naylor, et al.:[1]

(1) $$Ex = e^{\left(\mu_y + \frac{\sigma_y^2}{2}\right)}$$

(2) $$Vx = (E_x)^2 \left[e^{\sigma_y^2} - 1\right]$$

---

[1] Naylor, T., Balintfy, J., Burdick, D., and Chu, K., Computer Simulation Technique, Wiley & Sons, 1968, p. 100.

Let $Z = 2.57583$, where Z is the standard normal variate corresponding to a 0.005 probability of higher values.

(3) $\ln S_{max} = \mu_y + Z\sigma_y = \mu_y + 2.57583\ \sigma_y$

Given S_{max} and $R = \frac{\sqrt{Vx}}{E_x}$, one can find Ex:

(4) $\sigma_y = \sqrt{\ln\ (R^2 + 1)}$

(5) $\mu_y = \ln S_{max} - Z\ \sigma_y = \ln S_{max} - 2.57583\ \sigma_y$

(6) $Ex = e^{(\mu_y + \frac{\sigma_y^2}{2})}$

(7) $Ex = e^{\ln S_{max} - 2.57583\ \sigma_y + \frac{1}{2}\sigma_y^2}$

(8) $Ex = \left[S_{max}\right]\left[e^{\frac{1}{2}\sigma_y^2 - 2.57583\ \sigma_y}\right]$

TABLE 12, INVERTED GAMMA DISTRIBUTION

For the inverted gamma distribution the probability of a value being greater than the emission standard can be estimated by:

(1) $Pr\ (X > S_{max}) = Pr\ (\chi^2 < \frac{2a}{S_{max}}\ |\ f = 2B)$

First, determine B to the nearest integer by:

(2) $B = \frac{1}{RSD^2} + 2$

Next, determine the χ^2 (chi-square) value at the 0.005 confidence level for $f = 2B$, then

(3) $\frac{2a}{S_{max}} = \chi^2$

(4) $a = \frac{(\chi^2)\ (S_{max})}{2}$

Next determine μ_x by

(5) $\mu_x = \frac{a}{B - 1}$

APPENDIX B

DERIVATION OF FORMULA FOR TRUE AND MEASURED RSD

Definition of Terms:

C_m = measured sulfur content, lbs S/MMBtu
S_m = measured sulfur content, percent
H_m = measured heat content, Btu/lb
C_t = true sulfur content, lbs S/MMBtu
S_t = true sulfur content, percent
H_t = true heat content, Btu/lb
e_{ss} = error in sulfur measurement due to sampling
e_{sa} = error in sulfur measurement due to analysis
e_{hs} = error in heat measurement due to sampling
e_{ha} = error in heat measurement due to analysis
σ^2 = variance
σ = standard deviation
μ = mean
cov = covariance

Assume: $S_m = S_t + e_{ss} + e_{sa}$

$H_m = H_t + e_{hs} + e_{ha}$

$\mu_{e_{ss}} = \mu_{e_{sa}} = \mu_{e_{hs}} = \mu_{e_{ha}} = 0$

$\sigma^2_{S_m} = \sigma^2_{S_t} + \sigma^2_{e_{ss}} + \sigma^2_{e_{sa}}$ if it is assumed:

$\text{cov}(S_t, e_{ss}) = \text{cov}(S_t, e_{sa}) = \text{cov}(e_{ss}, e_{sa}) = 0$

$\sigma^2_{H_m} = \sigma^2_{H_t} + \sigma^2_{e_{hs}} = \sigma^2_{e_{ha}}$ if it is assumed:

$\text{cov}(H_t, e_{hs}) = \text{cov}(H_t, e_{ha}) = \text{cov}(e_{hs}, e_{ha}) = 0$

Then:

$$\mathrm{VAR}\left(\frac{S_t}{H_t}\right) \simeq \left(\frac{\mu_{S_t}}{\mu_{H_t}}\right)^2 \left[\frac{\sigma^2_{S_t}}{\mu^2_{S_t}} + \frac{\sigma^2_{H_t}}{\mu^2_{S_t}} - \frac{2\ \mathrm{cov}\ (S_t, H_t)}{\mu_{S_t}\ \mu_{H_t}}\right]$$

$$\mathrm{VAR}\left(\frac{S_m}{H_m}\right) \simeq \left(\frac{\mu_{S_m}}{\mu_{H_m}}\right)^2 \left[\frac{\sigma^2_{S_m}}{\mu^2_{S_m}} + \frac{\sigma^2_{H_m}}{\mu^2_{H_m}} - \frac{2\ \mathrm{cov}\ (S_m, H_m)}{\mu_{S_m}\ \mu_{H_m}}\right]$$

$$\mathrm{Ex\ VAR}\left(\frac{S_m}{H_m}\right) \simeq \left(\frac{\mu_{S_t}}{\mu_{H_t}}\right)^2 \left[\frac{\sigma^2_{S_t} + \sigma^2_{e_{ss}} + \sigma^2_{e_{sa}}}{\mu^2_{S_t}} + \frac{\sigma^2_{H_t} + \sigma^2_{e_{hs}} + \sigma^2_{e_{ha}}}{\mu^2_{H_t}} - \frac{2\ \mathrm{cov}\ (S_m, H_m)}{\mu_{S_t}\ \mu_{H_t}}\right]$$

Because $\mathrm{Ex}\ (\mu_{S_t}) = \mathrm{Ex}\ (\mu_{S_m})$ and,

$\mathrm{Ex}\ (\mu_{H_t}) = \mathrm{Ex}\ (\mu_{H_m})$

Assume: $\mathrm{Cov}\ (S_t, H_t) = \mathrm{Cov}\ (S_m, H_m)$

Then:

$$\mathrm{Ex\ VAR}\left(\frac{S_m}{H_m}\right) - \mathrm{VAR}\left(\frac{S_t}{H_t}\right) = \left(\frac{\mu_{S_t}}{\mu_{H_t}}\right)^2 \left[\frac{\sigma^2_{e_{ss}} + \sigma^2_{e_{sa}}}{\mu^2_{S_t}} + \frac{\sigma^2_{e_{ht}} + \sigma^2_{e_{ha}}}{\mu^2_{H_t}}\right]$$

$$= \left(\frac{\mu_{S_t}}{\mu_{H_t}}\right)^2 \left[R^2_{ss} + R^2_{sa} + R^2_{hs} + R^2_{ha}\right]$$

where:

R_{ss} = RSD of error in sulfur measurement due to sampling
R_{sa} = RSD of error in sulfur measurement due to analysis
R_{hs} = RSD of error in heat measurement due to sampling
R_{ha} = RSD of error in heat measurement due to analysis

Dividing by $\left(\frac{\mu_{S_t}}{\mu_{H_t}}\right)^2$ yields

$$RSD_m^2 - RSD_t^2 = R_{ss}^2 + R_{sa}^2 + R_{hs}^2 + R_{ha}^2$$

where RSD_m = RSD as measured
RSD_t = True RSD

APPENDIX C

COAL SULFUR ANALYSES DATA
DATA BASE TAPE FORMAT

Field Number (1)	Field Position From - To (2)	Field Size[1] (3)	Field Descriptions (4)
1	1- 2	XX	FIPS State Code[2]
2	3- 3	X	State Modifier[3]
3	4- 6	XXX	FIPS County Code[2]
4	7- 8	XX	USBM Producing District
5	9- 13	XXXXX	Company Code
6	14- 18	XXXXX	Supplier Code
7	19- 23	XXXXX	Mine Code Number
8	24- 28	XXXXX	Town Code Number[4]
9	29- 34	XXXXXX	Bed Code Number[5]
10	35- 35	X	Rank of Coal[6]
11	36- 36	X	Mining Method[7]
12	37- 37	X	Preparation Code[8]
13	38- 43	XXXXXX	Date of Sample[9]
14	44- 49	XXXXXX	Sequence Number[10]
15	50- 55	XXXXXX	Core Hole Number
16	56- 56	X	Method of Sampling[11]
17	57- 57	X	Type of Sample[12]
18	58- 64	XXXXXXX	Tonnage Sampled
19	65- 65	X	Flag for Estimated Tonnage[13]
20	66- 69	XX‸XX	Moisture Content, Percent, as Received
21	70- 73	XX‸XX	Volatile Matter, Percent, as Received
22	74- 77	XX‸XX	Fixed Carbon, Percent, as Received
23	78- 81	XX‸XX	Ash Content, Percent, as Received
24	82- 85	XX‸XX	Sulfur Content, Percent, as Received
25	86- 90	XXXXX	Heat Content, Btu per Pound, as Received
26	91- 94	XXXX	Ash Fusion Temperature, °F.
27	95- 98	XX‸XX	SO_2 Emissions, lbs SO_2/MMBtu[14]

Field Number (1)	Field Position From - To (2)	Field Size[1/] (3)	Field Descriptions (4)
28	99-102	XX$_\wedge$XX	Volatile Matter, Percent, Dry
29	103-106	XX$_\wedge$XX	Fixed Carbon, Percent, Dry
30	107-110	XX$_\wedge$XX	Ash Content, Percent, Dry
31	111-114	XX$_\wedge$XX	Sulfur Content, Percent, Dry
32	115-119	XXXXX	Heat Content, Btu per Pound, Dry
33	120-123	XX$_\wedge$XX	SO_2 Emissions, As Reported, lbs SO_2/MMBtu[15/]
34	124-125	XX	Size Code[16/]

1/ $\wedge$ = implied decimal.

2/ U.S. Department of Commerce, National Bureau of Standards, Federal Information Processing Standards Publication 601, November 1, 1968 (34 pages).

3/ For Kentucky 1 = Eastern, 2 = Western; For Pennsylvania 1 = Anthracite, 2 = Bituminous; all others = 0.

4/ CODES beginning with 0 are equivalent to four-digit codes used by USBM in attachment to analytical Data Tape documentation. Codes beginning with 1 represent towns not listed by USBM.

5/ First Digit: 0 means an equivalent USBM code exists, 1 means no USBM code exists and a special code was assigned.
Digits 2-4: equivalent to USBM bed codes.
Digits 5-6: identifiers for local bed names used for the same bed.

6/ 1 = lignite, 2 = subbituminous, 3 = bituminous, 4 = anthracite.

7/ 1 = underground, 2 = surface, 3 = surface-auger, 4 = underground-surface, 9 = unknown.

8/ 1 = Raw, 2 = washed or cleaned, 3 = partially washed or cleaned, 9 = unknown.

9/ Year-month-day.

10/ Railroad car, train, barge, or sample number.

11/ 1 = automatic sample--ASTM; 2 = hand sample--ASTM; 3 = automatic sample--non-ASTM; 4 = hand sample--non-ASTM; 9 = unknown.

12/ 1 = core sample, 2 = as mined, 3 = as shipped, 4 = as delivered, 5 = as burned, 9 = unknown.

13/ 0 = measured tonnage, 1 = estimated based on number of railroad cars, barges, etc

14/ Calculated based on sulfur and heat contents and assuming that 95 percent by weight of the sulfur present in the coal is released as sulfur dioxide.

15/ Calculated and reported by coal consuming company.

16/ Bureau of Mines data only.

APPENDIX D

STACK MONITORING DATA
DATA BASE TAPE FORMAT

Field Number (1)	Field Position From - To (2)	Field Size[1/] (3)	Field Descriptions (4)
1	1- 5	XXXXX	Company Code
2	6- 8	XXX	Generating Unit Number
3	9- 12	XXXX	Unit Size (MW)
4	13- 14	XX	FGD Unit Number
5	15- 20	XXXXXX	Date[2/]
6	21- 24	XXXX	Time
7	25- 28	XXXX	Gross Load (MWHr.)
8	29- 32	XXX$_{\wedge}$X	Coal Flow (Thousand Pounds/Time Period)
9	33- 37	XXXX$_{\wedge}$X	Inlet SO$_2$ (ppm Wet)
10	38- 40	XX$_{\wedge}$X	Inlet O$_2$ (percent)
11	41- 43	XX$_{\wedge}$X	Inlet H$_2$O (percent)
12	44- 47	XX$_{\wedge}$XX	Inlet SO$_2$ (lbs/MMBtu)
13	48- 52	XXXX$_{\wedge}$X	Outlet SO$_2$ (ppm Wet)
14	53- 55	XX$_{\wedge}$X	Outlet O$_2$ (percent)
15	56- 58	XX$_{\wedge}$X	Outlet H$_2$O (percent)
16	59- 62	XX$_{\wedge}$XX	Outlet SO$_2$ (lbs/MMBtu)
17	63- 65	XX$_{\wedge}$X	Efficiency[3/]
18	66- 71	XXXX$_{\wedge}$XX	Outlet SO$_2$; 1 Hr Avg. (Kg./Hr)
19	72- 77	XXXX$_{\wedge}$XX	Outlet SO$_2$; 3 Hr Avg. (Kg./Hr)
20	78- 82	X$_{\wedge}$XXXX	Outlet Wt. Sulfur; 3 Hr Running Avg. (percent
21	83- 88	XXXX$_{\wedge}$XX	Outlet Wt. Sulfur; 3 Hr Running Avg. (lbs)
22	89- 91	X$_{\wedge}$XX	Outlet SO$_2$; 1 Hr Avg. (lbs/MMBtu)
23	92- 94	X$_{\wedge}$XX	Outlet SO$_2$; 3 Hr Avg. (lbs/MMBtu)
24	95- 97	X$_{\wedge}$XX	Outlet SO$_2$; 24 Hr Avg. (lbs/MMBtu)
25	98-100	XX$_{\wedge}$X	Outlet O$_2$; 1 Hr Avg. (percent)
26	101-103	XX$_{\wedge}$X	Outlet O$_2$; 3 Hr Avg. (percent)
27	104-106	XX$_{\wedge}$X	Outlet O$_2$; 24 Hr Avg. (percent)

Field Number (1)	Field Position From - To (2)	Field Size[1/] (3)	Field Descriptions (4)
28	107-111	XXXXX	Coal Source No. 1, Mine Code Number
29	112-116	XXXXX	Coal Source No. 2, Mine Code Number
30	117-121	XXXXX	Coal Source No. 3, Mine Code Number
31	122-126	XXXXX	Coal Source No. 4, Mine Code Number

[1/] $\wedge$ = implied decimal.

[2/] Year-Month-Day

[3/] $\text{Efficiency} = \frac{\text{Inlet } SO_2 - \text{Outlet } SO_2}{\text{Inlet } SO_2}$

APPENDIX E

INDEX TO MINE LOCATIONS AND SEAMS PRODUCED

				SEAM(S) PRODUCED		
Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	Reference Code (5)	Name (6)	Composite Of Seams (7)
10001		KY E	Knott	010401	Hazard No. 7	
10002		KY E	McCreary	015109	Jellico	
10003		KY E	Clay Laurel	100001	Hazard No. 4 Horse Creek Lily	013520 021202 021203
10004		IL	Peoria	090002	No. 6	
10005		MT	Rosebud	080800	Rosebud	
10006		WY	Campbell	095100	Smith & Roland	
10007		WY	Campbell	100003	Wyodak	-
10008	10006 10007	WY WY	Campbell Campbell	100002	Smith & Roland Wyodak	095100 -
10009		KY W	Muhlenburg	048913	No. 9*	
10010		KS	Crawford	049202	Bevier	
10011		MO	Howard	049202	Bevier	
10012		MO	Audrain	049008	Mulky	
10013		MO	Howard	049202	Bevier	
10014		MO	Randolph	049202	Bevier	
10015		MO	Randolph	098700	Bevier and Wheeler	
10016		KY E	Pike	100004	Dorothy Thacker	012105 015119
10017		MD	Allegany	100032	Pittsburgh Sewickley	003608 002902
10018		KY W	Webster	012704	No. 9	
10019		KY W	Hopkins	012704	No. 9	
10020		KY W	Union	048415	No. 11*	
10021		KY W	Union	012704	No. 9	
10022		KY E	Martin	100005	Stockton No. 5 Block Clarion	010301 008402 008701
10023		WV	Logan	015104	Cedar Grove	
10024		IL	Perry	090002	No. 5 and 6	
10025		KY E	Perry	100006	Hazard No. 5 Hazard No. 5A Hazard No. 7 Hazard No. 9	012103 011108 010401 009601
10026		KY E	Rockcastle	015108	Elkhorn No. 3	
10027		KY E	Clay	021202	Horse Creek	
10028		KY E	Breathitt	011108	Hazard No. 5A	
10029		WV	Monongalia	003604	Pittsburgh	
10030		OH	Harrison	007402	Lower Freeport	
10031		WV	Grant	007102	Upper Freeport	
10032		TN	Marion Sequatchie	028601	Sewanee	

				SEAM(S) PRODUCED		
Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	Reference Code (5)	Name (6)	Composite Of Seams (7)
10033		KY E	Johnson	008402	No. 5 Block	
10034		KY E	Elliot Lawrence Martin	016200	Mudslip	
10035		IL	Christian	048408	No. 6	
10036		IL	Perry	090002	No. 5 and 6	
10037		MT	Bighorn	069802	Dietz No. 1	
10038		WY	Carbon	036555	No. 25	
10039		WY	Carbon	081700	Hanna No. 2	
10040		WY	Sheridan	078300	Monarch	
10041		WY	Sweetwater	100007	Deadman Bed	
10042		IL	Macoupin	048408	No. 6	
10043		IL	Fulton	048905	No. 5	
10044		ND	Mercer	056100	Scranton	
10045		IL	St. Clair	048408	No. 6	
10046		IL	Randolph	048408	No. 6	
10047		WV	Grant	100008	Bakerston Upper Freeport	006305 007102
10048		KY E	Bell	100009	Hignite Red Spring + 9 others	012601 010406 -
10049		KY W	Ohio	090000	No. 9* and 11*	
10050		KY E	Whitley	015703	Blue Gem	
10051		KY E	Knox	015703	Blue Gem	
10052		KY W	Muhlenburg	100010	No. 11 No. 12 No. 9	048415 048306 048913
10053		PA	Armstrong	095203	Lower & Upper Freeport	
10054		PA	Indiana	007102	Upper Freeport	
10055		PA	Indiana	007102	Upper Freeport	
10056		KY W	Unknown	048913	No. 9*	
10057		CO	Routt	009900	Fish Creek	
10058		IL	Douglas	048408	No. 6	
10059		ND	Mercer	056901	Beulah-Zap	
10060		ND	Bowman	056400	Harmon	
10061		WY	Campbell	092600	Wyodak-Anderson	
10062		IA	Monroe	051700	Lucas County No. 5	
10063		MO	Randolph	049202	Bevier	
10064		MO	Macon	049202	Bevier	
10065		IA	Lucas	051700	Lucas County No. 5	
10066		MO	Putnam	048414	Lexington	
10067		IA	Mahaska	053004	Lower Ford	
10068		IA	Mahaska		Unknown	
10069		IA	Mahaska		Unknown	
10070		IA	Marion		Unknown	

Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	SEAM(S) PRODUCED		
				Reference Code (5)	Name (6)	Composite Of Seams (7)
10071		AZ	Navajo	100011	Green	050100
					Red	050200
					Blue	050300
10072		KY E	Perry	100012	Hazard No. 5A	011108
					Hazard No. 7	010401
					and others	-
10073		KY E	Floyd	100013	Elkhorn No. 1	015704
					Elkhorn No. 2	015402
					Fire Clay	013504
					Van Lear	015126
10074	10072	KY E	Perry	100014	Hazard No. 5A	011108
	10073		Floyd		Hazard No. 7	010401
					Elkhorn No. 1	015704
					Elkhorn No. 2	015402
					Fire Clay	013504
					Van Lear	015126
					and others	-
10075		KY W	Henderson	048913	No. 9*	
10076		WY	Campbell	095100	Smith & Roland	
10077		WY	Carbon	100017	No. 80	039900
					No. 82	040100
10078		CO	Weld	076800	Laramie No. 3	
10079		CO	Jackson	004900	Sudduth	
10080		PA	Clearfield	100015	Lower Kittanning	008419
					Middle Kittanning	008002
					Upper Kittanning	007603
					Lower Freeport	007402
					Upper Freeport	007102
10081		PA	Clearfield	100015	Lower Kittanning	008419
					Middle Kittanning	008002
					Upper Kittanning	007603
					Lower Freeport	007402
					Upper Freeport	007102
10082		PA	Armstrong	100016	Lower Kittanning	008419
					Middle Kittanning	008002
					Upper Kittanning	007603
10083		WV	Marion	003604	Pittsburgh	
10084		WV	Marion	003604	Pittsburgh	
10085		PA	Clearfield	100016	Lower Kittanning	008419
					Middle Kittanning	008002
					Upper Kittanning	007603
10086		PA	Fayette	100016	Lower Freeport	007402
					Upper Freeport	007102
					Waynesburg	002302
10087		KY E.	Martin	016811	Pond Creek	
10088		CO	Routt	075000	Wadge	
10089		CO	Moffat	076900	Collom	
10090		CO	Moffat	075701	F	

				SEAM(S) PRODUCED		
Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	Reference Code (5)	Name (6)	Composite Of Seams (7)
10091		CO	Las Animas	074502	Robinson	
10092		IL	Perry	048408	No. 6	
10093		IL	Perry	090002	No. 5 and 6	
10094		IL	Jackson	048408	No. 6	
10095		IL	Macoupin	048408	No. 6	
10096		KY W.	Ohio	050609	No. 6*	
10097		IL	Perry	090002	No. 5 and 6	
10098		NM	McKinley	047300	Green	
10099		WY	Carbon	039800	No. 65	
10100		IN	Pike	100018	V	048911
					Lower Millersburg	048304
10101		IL	Franklin	048408	No. 6	
10102		IL	Franklin	048408	No. 6	
10103		IL	Jefferson	048408	No. 6	
10104		IL	Jefferson	048408	No. 6	
10105		IL	St. Clair	048408	No. 6	
10106		CO	Moffat	100021	Yampa Field	-
10107		UT	Carbon	100022	Hiawatha	084601
					Wattis	023600
10108		VA	Buchanan	100023	Glamorgam	018505
					Splash Dam	021002
					Blair	017701
					Hagy	019502
10109		IL	Randolph	048408	No. 6	
10110		KY E	Unknown	100024	Unknown	
10111		KY E	Perry	100025	Unknown	
10112		KY E	Harlan	100026	Unknown	
10113		OH	Muskingum		Unknown	
10114		OH	Muskingum	003607	No. 8	
10115		OH	Coshocton	008013	No. 6	
10116		OH	Coshocton	008013	No. 6	
10117		OH	Coshocton	008013	No. 6	
10118		OH	Guernsey Belmont		Unknown	
10119		OH	Perry	008002	Middle Kittanning	
10120		OH	Coshocton	002302	Waynesburg	
10121		OH	Tuscarawas	100027	Lower Kittanning	008404
					Middle Kittanning	008010
10122		OH	Tuscarawas	008404	Lower Kittanning	
10123		OH	Coshocton	008424	No. 5	
10124		OH	Coshocton	008424	No. 5	
10125		OH	Tuscarawas	008404	Lower Kittanning	
10126		OH	Coshocton		Unknown	
10127		OH	Jackson Vinton	100028	Brookville	009501
					No. 4A	008705
					No. 5	008424
					No. 6	008013
10128		OH	Tuscarawas	100029	No. 7	007117
					No. 7A	007007

				SEAM(S) PRODUCED		
Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	Reference Code (5)	Name (6)	Composite Of Seams (7)
10129		OH	Harrison Belmont		Unknown	
10130		OH	Jefferson	003604	Pittsburgh	
10131		OH	Coshocton		Unknown	
10132		OH	Unknown		Unknown	
10133		OH	Muskingum	008002	Middle Kittanning	
10134		OH	Tuscarawas	008002	Middle Kittanning	
10135		OH	Unknown		Unknown	
10136		OH	Unknown		Unknown	
10137		OH	Coshocton	008013	No. 6	
10138		OH	Perry		Unknown	
10139		OH	Unknown		Unknown	
10140		OH	Unknown		Unknown	
10141		OH	Unknown		Unknown	
10142		OH	Vinton	008701	Clarion	
10143		OH	Vinton		Unknown	
10144		OH	Hocking	007406	No. 6A	
10145		OH	Vinton		Unknown	
10146		OH	Hocking	008013	No. 6	
10147		OH	Perry	008013	No. 6	
10148		OH	Vinton	088501	Clarion & L. Kittanning	
10149		OH	Hocking		Unknown	
10150		OH	Hocking Vinton	088701	Brookville & M. Kittanning	
10151		OH	Unknown		Unknown	
10152		OH	Unknown		Unknown	
10153		OH	Unknown		Unknown	
10154		OH	Unknown		Unknown	
10155		OH	Unknown		Unknown	
10156		OH	Unknown		Unknown	
10157		OH	Unknown		Unknown	
10158		OH	Guernsey Belmont	085801	Meigs Creek & Waynesburg	
10159		OH	Muskingum		Unknown	
10160		OH	Perry		Unknown	
10161		OH	Coshocton		Unknown	
10162		OH	Perry		Unknown	
10163		OH	Tuscarawas		Unknown	
10164		OH	Morgan		Unknown	
10165		OH	Jackson		Unknown	
10166		OH	Unknown		Unknown	
10167		OH	Vinton		Unknown	
10168		OH	Unknown		Unknown	
10169		OH	Unknown		Unknown	
10170		OH	Unknown		Unknown	
10171		OH	Muskingum	008002	Middle Kittanning	
10172		OH	Unknown		Unknown	

				SEAM(S) PRODUCED		
Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	Reference Code (5)	Name (6)	Composite Of Seams (7)
10173		OH	Perry		Unknown	
10174		OH	Coshocton	092801	L. & M. Kittanning	
10175	10169	OH	Unknown	100031	Unknown	
	10138	OH	Perry		Unknown	
	10139	OH	Unknown		Unknown	
	10113	OH	Muskingum		Unknown	
	10114	OH	Muskingum		No. 8	003607
	10118	OH	Guernsey Belmont		Unknown	
	10157	OH	Unknown		Unknown	
	10130	OH	Jefferson		Pittsburgh	003604
	10161	OH	Coshocton		Unknown	
	10119	OH	Perry		Middle Kittanning	008002
	10131	OH	Coshocton		Unknown	
	10132	OH	Unknown		Unknown	
	10120	OH	Coshocton		Waynesburg	002302
	10122	OH	Tuscarawas		Lower Kittanning	008404
	10140	OH	Unknown		Unknown	
	10170	OH	Unknown		Unknown	
	10133	OH	Muskingum		Middle Kittanning	008002
	10123	OH	Coshocton		No. 5	008424
	10124	OH	Coshocton		No. 5	008424
	10135	OH	Unknown		Unknown	
	10159	OH	Muskingum		Unknown	
	10158	OH	Guernsey Belmont		Meigs Creek & Waynesburg	085801
	10125	OH	Tuscarawas		Lower Kittanning	008404
	10141	OH	Unknown		Unknown	
	10160	OH	Perry		Unknown	
	10134	OH	Tuscarawas		Middle Kittanning	008002
	10126	OH	Coshocton		Unknown	
	10168	OH	Unknown		Unknown	
	10127	OH	Jackson Vinton		Brookville No. 4A No. 5 No. 6	009501 008705 008424 008013
	10128	OH	Tuscarawas		No. 7 No. 7A	007117 007007
	10162	OH	Perry		Unknown	
	10116	OH	Coshocton		No. 6	008013
	10117	OH	Coshocton		No. 6	008013
	10115	OH	Coshocton		No. 8	003607
	10137	OH	Coshocton		No. 6	
	10163	OH	Tuscarawas		Unknown	
	10129	OH	Harrison Belmont		Unknown	
	10136	OH	Unknown		Unknown	
	10164	OH	Morgan		Unknown	

Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	SEAM(S) PRODUCED Reference Code (5)	Name (6)	Composite Of Seams (7)
10176	10113	OH	Muskingum	100031	Unknown	
	10142	OH	Vinton		Clarion	008701
	10157	OH	Unknown		Unknown	
	10143	OH	Vinton		Unknown	
	10144	OH	Hocking		No. 6A	007406
	10119	OH	Perry		Middle Kittanning	008002
	10156	OH	Unknown		Unknown	
	10145	OH	Vinton		Unknown	
	10159	OH	Muskingum		Unknown	
	10155	OH	Unknown		Unknown	
	10152	OH	Unknown		Unknown	
	10160	OH	Perry		Unknown	
	10171	OH	Muskingum		Middle Kittanning	008002
	10146	OH	Hocking		No. 6	008013
	10151	OH	Unknown		Unknown	
	10127	OH	Jackson Vinton		Brookville No. 4A No. 5 No. 6	009501 008705 008424 008013
	10154	OH	Unknown		Unknown	
	10148	OH	Vinton		Clarion & L. Kittanning	088501
	10153	OH	Unknown		Unknown	
10177	10157	OH	Unknown	100031	Unknown	
	10142	OH	Vinton		Clarion	008701
	10143	OH	Vinton		Unknown	
	10144	OH	Hocking		No. 6A	007406
	10174	OH	Coshocton		L. & M. Kittanning	092801
	10165	OH	Jackson		Unknown	
	10119	OH	Perry		Middle Kittanning	008002
	10173	OH	Perry		Unknown	
	10149	OH	Hocking		Unknown	
	10145	OH	Vinton		Unknown	
	10167	OH	Vinton		Unknown	
	10172	OH	Unknown		Unknown	
	10160	OH	Perry		Unknown	
	10146	OH	Hocking		No. 6	008013
	10150	OH	Hocking Vinton		Brookville & M. Kittanning	088701
	10147	OH	Perry		No. 6	008013
	10166	OH	Unknown		Unknown	
	10148	OH	Vinton		Clarion & L. Kittanning	088501

Mine Code (1)	Blend of Mines (2)	State (3)	County (4)	Seams(s) Produced Reference Code (5)	Name (6)	Composite of Seams (7)
10178		MT	Big Horn	100033	Rosebud & McKay	093100
					Robinson	051700
10179		ND	Oliver	100034	Hagel	
					Top	
10180		--	Unknown		Unknown	
10181		--	Unknown		Unknown	
10182		KY E	Unknown		Unknown	
10183		KY W	Muhlenburg	100010	No. 11*	048415
					No. 12	048306
					No. 9	048913
10184		IN	Warrick	048305	No. VI	
10185		IN	Clay	050202	No. III	
10186		KY W	Muhlenburg	100035	No. 11*	048415
					No. 12	048306
					No. 13	048209
10187		IN	Sullivan	100036	No. V	048905
					No. VI	048305
					No. VII	048006
10188		IL	Wabash	048905	No. 5	
10189		IN	Warrick	048905	No. V	
10190		IL	Fulton	049608	No. 2	
10191		KY E	Knott	011108	Hazard No. 5-A	
10192		OH	Belmont	085802	No. 9 & No. 11	
10193		WV	Kanawha	008402	No. 5 Block	
10194		MT	Wibaux		Unknown	
10195		MT	Richland		Unknown	
10196		MT	Richland	034500	Pust	
10197		MT	Powder River		Unknown	
10198		MT	Custer		Unknown	
10199		MT	Custer		Unknown	
10200		MT	Custer		Unknown	
10201		MT	Rosebud		Unknown	
10202		MT	Rosebud	093100	Rosebud & McKay	
10203		MT	Big Horn	093100	Rosebud & McKay	
10204		PA	Somerset		Unknown	
10205		PA	Jefferson		Unknown	
10206		WV	Barbour		Unknown	
10207		PA	Clearfield		Unknown	
10208		PA	Cambria Indiana		Unknown	
10209		PA	Somerset		Unknown	
10210		MD	Garrett		Unknown	
10211		PA	Somerset		Unknown	
10212		PA	Armstrong		Unknown	

				Seams(s) Produced		
Mine Code (1)	Blend of Mines (2)	State (3)	County (4)	Reference Code (5)	Name (6)	Composite of Seams (7)
10213		PA	Clarion		Unknown	
10214		PA	Clarion		Unknown	
10215		PA	Clearfield		Unknown	
10216		PA	Somerset		Unknown	
10217		PA	Cambria		Unknown	
10218		PA	Clearfield		Unknown	
10219		PA	Centre		Unknown	
10220		WV	Mineral		Unknown	
10221		PA	Clearfield		Unknown	
10222		PA	Somerset		Unknown	
10223		PA	Somerset		Unknown	
10224		PA	Somerset		Unknown	
10225		PA	Jefferson		Unknown	
10226		PA	Clearfield		Unknown	
10227		PA	Indiana		Unknown	
10228		WV	Boone		Unknown	
10229		PA	Somerset		Unknown	
10230		WV	Mineral		Unknown	
10231		PA	Indiana		Unknown	
10232		--	Unknown		Unknown	
10233		--	Unknown		Unknown	
10234		--	Unknown		Unknown	
10235		PA	Clearfield		Unknown	
10236		PA	Jefferson		Unknown	
10237		PA	Armstrong		Unknown	
10238		PA	Somerset		Unknown	
10239		MD	Allegheny		Unknown	
10240		PA	Clearfield		Unknown	
10241		PA	Jefferson		Unknown	
10242		PA	Clearfield		Unknown	
10243		WV	Harrison		Unknown	
10244		PA	Indiana		Unknown	
10245		--	Unknown		Unknown	
10246		PA	Somerset		Unknown	
10247		--	Unknown		Unknown	
10248		PA	Clarion		Unknown	
10249		PA	Centre		Unknown	
10250		PA	Somerset		Unknown	
10251		PA	Elk		Unknown	
10252		PA	Somerset		Unknown	
10253		PA	Clearfield		Unknown	
10254		PA	Clearfield		Unknown	

Mine Code (1)	Blend of Mines (2)	State (3)	County (4)	Seams(s) Produced: Reference Code (5)	Seams(s) Produced: Name (6)	Seams(s) Produced: Composite of Seams (7)
10255		PA	Clearfield		Unknown	
10256		PA	Jefferson		Unknown	
10257		PA	Somerset		Unknown	
10258		PA	Cambria		Unknown	
10259		PA	Clearfield		Unknown	
10260		PA	Westmoreland		Unknown	
10261		WV	Randolph		Unknown	
10262		PA	Armstrong Indiana		Unknown Unknown	
10263		PA	Armstrong Indiana		Unknown Unknown	
10264		PA	Clearfield		Unknown	
10265		PA	Indiana		Unknown	
10266		PA	Somerset		Unknown	
10267		PA	Armstrong		Unknown	
10268		PA	Armstrong		Unknown	
10269		PA	Armstrong		Unknown	
10270		PA	Armstrong		Unknown	
10271		PA	Clearfield		Unknown	
10272		PA	Fayette		Unknown	
10273		PA	Somerset		Unknown	
10274		WV	Barbour		Unknown	
10275		PA	Somerset		Unknown	
10276		PA	Cambria		Unknown	
10277		--	Unknown		Unknown	
10278		PA	Armstrong Indiana		Unknown Unknown	
10279		PA	Cambria		Unknown	
10280		PA	Clearfield		Unknown	
10281		PA	Clearfield		Unknown	
10282		PA	Somerset		Unknown	
10283		PA	Jefferson		Unknown	
10284		PA	Jefferson		Unknown	
10285		PA	Jefferson		Unknown	
10286		WV	Preston		Unknown	
10287		PA	Somerset		Unknown	
10288		PA	Clearfield		Unknown	
10289		PA	Clearfield		Unknown	
10290		PA	Clearfield		Unknown	
10291		PA	Clearfield		Unknown	
10292		--	Unknown		Unknown	
10293		PA	Armstrong		Unknown	
10294		PA	Armstrong		Unknown	
10295		PA	Armstrong		Unknown	
10296		PA	Clearfield		Unknown	
10297		PA	Clearfield		Unknown	
10298		PA	Somerset		Unknown	

Mine Code (1)	Blend Of Mines (2)	State (3)	County (4)	Seams(s) Produced Reference Code (5)	Name (6)	Composite of Seams (7)
10299		PA	Clearfield		Unknown	
10300		PA	Clarion		Unknown	
10301		PA	Clearfield		Unknown	
10302		WV	Harrison		Unknown	
10303		PA	Clearfield		Unknown	
10304		PA	Somerset		Unknown	
10305		WV	Preston		Unknown	
10306		PA	Cambria		Unknown	
10307		PA	Jefferson		Unknown	
--			Clearfield		Unknown	
10308		WV	Barbour		Unknown	
10309		--	Unknown		Unknown	
10310		PA	Clearfield		Unknown	
10311		PA	Jefferson		Unknown	
10312		PA	Jefferson		Unknown	
10313		PA	Clearfield		Unknown	
10314		PA	Somerset		Unknown	
10315		PA	Clearfield		Unknown	
10316		PA	Jefferson		Unknown	
10317		PA	Jefferson		Unknown	
10318		PA	Jefferson		Unknown	
10319		PA	Clearfield		Unknown	
10320		PA	Clearfield		Unknown	
10321		PA	Indiana		Unknown	
10322		WV	Monongalia		Unknown	
10323		PA	Clearfield		Unknown	
10324		PA	Clearfield		Unknown	
10325		PA	Indiana		Unknown	
10326		PA	Somerset		Unknown	
10327		PA	Clearfield		Unknown	
10328		PA	Indiana		Unknown	
10329		PA	Clearfield		Unknown	
10330		KY E	Martin		Unknown	
10331		WV	Unknown		Unknown	

APPENDIX F

MAPS OF MINE LOCATIONS BY PRODUCING DISTRICT, STATE AND COUNTY

Definition of Bureau of Mines Bituminous Coal and Lignite Producing Districts

DISTRICT 1.—EASTERN PENNSYLVANIA

Pennsylvania

Armstrong County (part).—All mines east of Allegheny River, and those mines served by the Pittsburgh & Shawmut Railroad located on the west bank of the river.

Fayette County (part).—All mines located on and east of the line of Indian Creek Valley branch of the Baltimore & Ohio Railroad.

Indiana County (part).—All mines not served by the Saltsburg branch of the Pennsylvania Railroad.

Westmoreland County (part).—All mines served by the Pennsylvania Railroad from Torrance, east.

All mines in the following counties:

Bedford
Blair
Bradford
Cambria
Cameron
Centre
Clarion
Clearfield
Clinton
Elk
Forest
Fulton
Huntingdon
Jefferson
Lycoming
McKean
Mifflin
Potter
Somerset
Tioga

Maryland.—All mines in the State.

West Virginia.—All mines in the following counties:

Grant
Mineral
Tucker

DISTRICT 2.—WESTERN PENNSYLVANIA

Pennsylvania

Armstrong County (part).—All mines west of the Allegheny River except those mines served by the Pittsburgh & Shawmut Railroad.

Fayette County (part).—All mines except those on and east of the line of Indian Creek Valley branch of the Baltimore & Ohio Railroad.

Indiana County (part).—All mines served by the Saltsburg branch of the Pennsylvania Railroad.

Westmoreland County (part).—All mines except those served by the Pennsylvania Railroad from Torrance, east.

All mines in the following counties:

Allegheny
Beaver
Butler
Greene
Lawrence
Mercer
Venango
Washington

DISTRICT 3.—NORTHERN WEST VIRGINIA

West Virginia

Nicholas County (part).—All mines served by or north of the Baltimore & Ohio Railroad.

All mines in the following counties:

Barbour
Braxton
Calhoun
Doddridge
Gilmer
Harrison
Jackson
Lewis
Marion
Monongalia
Pleasants
Preston
Randolph
Ritchie
Roane
Taylor
Tyler
Upshur
Webster
Wetzel
Wirt
Wood

DISTRICT 4.—OHIO.—All mines in the State.

DISTRICT 5.—MICHIGAN.—All mines in the State.

DISTRICT 6.—PANHANDLE

West Virginia.—All mines in the following counties:

Brooke
Hancock
Marshall
Ohio

DISTRICT 7.—SOUTHERN NO. 1

West Virginia

Fayette County (part).—All mines east of Gauley River and all mines served by the Gauley River branch of the Chesapeake & Ohio Railroad and mines served by the Virginian Railway.

McDowell County (part).—All mines in that portion of the county served by the Dry Fork branch of the Norfolk & Western Railroad and east thereof.

Raleigh County (part).—All mines except those on the Coal River branch of the Chesapeake & Ohio Railroad and north thereof.

Wyoming County (part).—All mines in that portion served by the Gilbert branch of the Virginian Railway lying east of the mouth of Skin Fork of Guyandot River and in that portion served by the main line and the Glen Rogers branch of the Virginian Railway.

All mines in the following counties:

Greenbrier
Mercer
Monroe
Pocahontas
Summers

Virginia

Buchanan County (part).—All mines in that portion of the county served by the Richlands-Jewell Ridge branch of the Norfolk & Western Railroad and in that portion on the headwaters of Dismal Creek east of Lynn Camp Creek (a tributary of Dismal Creek).

Tazewell County (part).—All mines in those portions of the county served by the Dry Fork branch to Cedar Bluff and from Bluestone Junction to Boissevain branch of the Norfolk & Western Railroad and Richlands-Jewell Ridge branch of the Norfolk & Western Railroad.

All mines in the following counties:

Montgomery
Pulaski
Wythe
Giles
Craig

DISTRICT 8.—SOUTHERN NO. 2

West Virginia

Fayette County (part).—All mines west of the Gauley River except mines served by the Gauley River branch of the Chesapeake & Ohio Railroad.

McDowell County (part).—All mines west of and not served by the Dry Fork branch of the Norfolk & Western Railroad.

Nicholas County (part).—All mines in that part of the county south of and not served by the Baltimore & Ohio Railroad.

Raleigh County (part).—All mines on the Coal River branch of the Chesapeake & Ohio Railroad and north thereof.

Wyoming County (part).—All mines in that portion served by the Gilbert branch of the Virginian Railway and lying west of the mouth of Skin Fork of Guyandot River.

All mines in the following counties:

Boone
Cabell
Clay
Kanawha
Lincoln
Logan
Mason
Mingo
Putnam
Wayne

Virginia

Buchanan County (part).—All mines in the county, except in that portion on the headwaters of Dismal Creek, east of Lynn Camp Creek (a tributary of Dismal Creek) and in that portion served by the Richlands-Jewell Ridge branch of the Norfolk & Western Railroad.

Tazewell County (part).—All mines in the county except in those portions served by the Dry Fork branch of the Norfolk & Western Railroad and branch from Bluestone Junction to Boissevain of Norfolk & Western Railroad and Richlands Jewell Ridge branch of the Norfolk & Western Railroad.

All mines in the following counties:

Dickinson
Lee
Russell
Scott
Wise

Kentucky.—All mines in the following counties in eastern Kentucky:

Bell
Boyd
Breathitt
Carter
Clay
Elliott
Floyd
Greenup
Harlan
Jackson
Johnson
Knott
Knox
Laurel
Lawrence
Lee
Leslie
Letcher
McCreary
Magoffin
Martin
Morgan
Owsley
Perry
Pike
Rockcastle
Wayne
Whitley

Tennessee.—All mines in the following counties:

Anderson
Campbell
Claiborne
Cumberland
Fentress
Morgan
Overton
Putnam
Roane
Scott

North Carolina.—All mines in the State.

DISTRICT 9.—WEST KENTUCKY

Kentucky.—All mines in the following counties in western Kentucky:

Butler
Christian
Crittenden
Daviess
Hancock
Henderson
Hopkins
Logan
McLean
Muhlenberg
Ohio
Simpson
Todd
Union
Warren
Webster

DISTRICT 10.—ILLINOIS.—All mines in the State.

DISTRICT 11.—INDIANA.—All mines in the State.

DISTRICT 12.—IOWA.—All mines in the State.

DISTRICT 13.—SOUTHEASTERN

Alabama.—All mines in the State.

Georgia.—All mines in the following counties:

Dade
Walker

Tennessee.—All mines in the following counties:

Bledsoe
Grundy
Hamilton
Marion
McMinn
Rhea
Sequatchie
Van Buren
Warren
White

DISTRICT 14.—ARKANSAS-OKLAHOMA

Arkansas.—All mines in the State.

Oklahoma.—All mines in the following counties:

Haskell
Le Flore
Sequoyah

DISTRICT 15.—SOUTHWESTERN

Kansas.—All mines in the State.

Missouri.—All mines in the State.

Oklahoma.—All mines in the following counties:

Coal
Craig
Latimer
Muskogee
Okmulgee
Pittsburg
Rogers
Tulsa
Wagoner

DISTRICT 16.—NORTHERN COLORADO

All mines in the following counties in the State:

Adams
Arapahoe
Boulder
Douglas
Elbert
El Paso
Jackson
Jefferson
Larimer
Weld

DISTRICT 17.—SOUTHERN COLORADO

Colorado.—All mines except those included in District 16.

New Mexico.—All mines except those included in District 18.

DISTRICT 18.—NEW MEXICO

New Mexico.—All mines in the following counties:

Grant
Lincoln
McKinley
Rio Arriba
Sandoval
San Juan
San Miguel
Santa Fe
Socorro

Arizona.—All mines in the State.

California.—All mines in the State.

DISTRICT 19.—WYOMING

Wyoming.—All mines in the State.

Idaho.—All mines in the State.

DISTRICT 20.—UTAH.—All mines in the State.

DISTRICT 21.—NORTH DAKOTA-SOUTH DAKOTA.—

All mines in North Dakota and South Dakota.

DISTRICT 22.—MONTANA.—All mines in the State.

DISTRICT 23.—WASHINGTON

Washington.—All mines in the State.

Oregon.—All mines in the State.

Alaska.—All mines in the State.

Source: Bureau of Mines.

PRODUCING DISTRICTS 1 AND 2

MARYLAND (District 1) 3 Mines WEST VIRGINIA (District 1) 4 Mines
PENNSYLVANIA (District 1) 109 Mines PENNSYLVANIA (District 2) 3 Mines

Clarion (4 Mines)
10213
10214
10248
10300

Armstrong (14 Mines)
10212
10237
10262
10263
10267
10268
10053
10269
10270
10278
10293
10294
10295
10082

Indiana (9 Mines)
10054
10055
10227
10231
10244*
10265
10321*
10325
10328

Westmoreland (1 Mine)
10260

Fayette (2 Mines)
10086
10272*

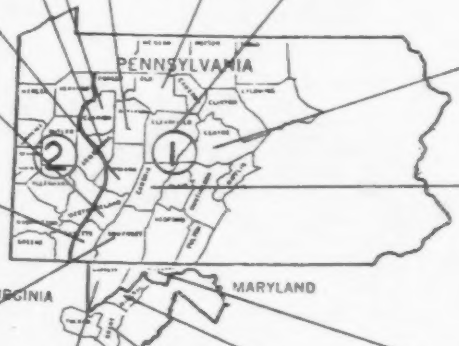
Jefferson (14 Mines)
10205
10225
10236
10241
10256
10283
10284
10285
10307
10311
10312
10316
10317
10318

Elk (1 Mine)
10251

Clearfield (37 Mines)
10080
10081
10085
10207
10215
10218
10221
10226
10240
10242
10253
10254
10235
10255
10259
10264
10271
10280
10281
10288
10289
10291
10296
10297
10290
10299
10301
10303
10310
10313
10315
10319
10320
10324
10327
10329
10323

Centre (2 Mines)
10219
10249

Cambria (6 Mines)
10208
10217
10258
10276
10279
10306



Somerset (22 Mines)
10204
10209
10211
10216
10222
10223
10224
10229
10238
10246
10250
10252
10257
10266
10273
10275
10282
10287
10298
10304
10314
10326

Garrett (1 Mine)
10210

Grant (2 Mines)
10031
10047

Allegany (2 Mines)
10017
10239

Mineral (2 Mines)
10220
10230

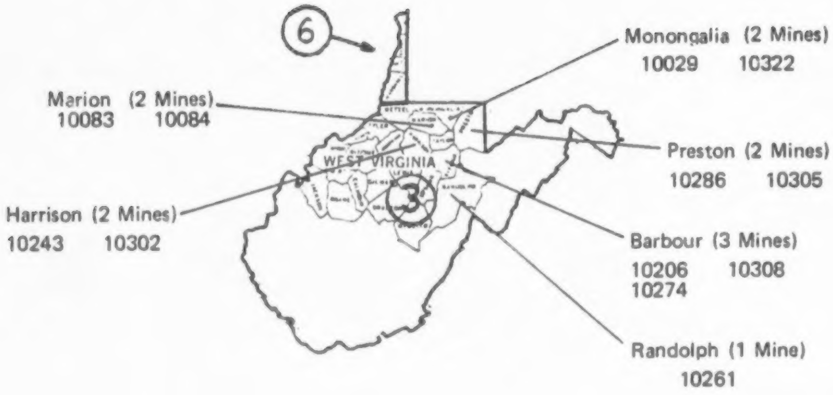
Unknown (9 Mines)
10232
10233
10234
10245
10247
10277
10292
10309
10331

* District 2

PRODUCING DISTRICTS 3 AND 6

WEST VIRGINIA

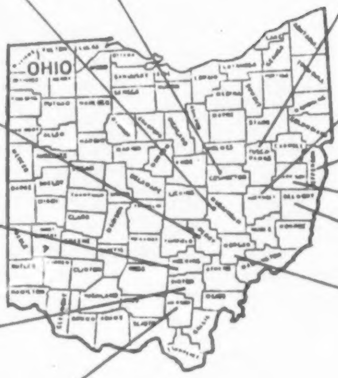
District 3	12 Mines
District 6	0 Mines



6
3
WEST VIRGINIA
Monongalia (2 Mines)
10029 10322
Marion (2 Mines)
10083 10084
Preston (2 Mines)
10286 10305
Harrison (2 Mines)
10243 10302
Barbour (3 Mines)
10206 10308
10274
Randolph (1 Mine)
10261

PRODUCING DISTRICT 4

OHIO 64 Mines



Coshocton (11 Mines)
10115 10126
10116 10131
10117 10137
10120 10161
10123 10174
10124

Muskingum (5 Mines)
10113 10159
10114 10171
10133

Perry (6 Mines)
10119 10160
10138 10162
10147 10173

Hocking (4 Mines)
10144 10149
10146 10150

Vinton (5 Mines)
10142 10148
10143 10167
10145

Jackson (2 Mines)
10127 10165

Tuscarawas (6 Mines)
10121 10128
10122 10134
10125 10163

Guernsey (2 Mines)
10118 10158

Jefferson (1 Mine)
10130

Harrison (2 Mines)
10030 10129

Belmont (1 Mine)
10192

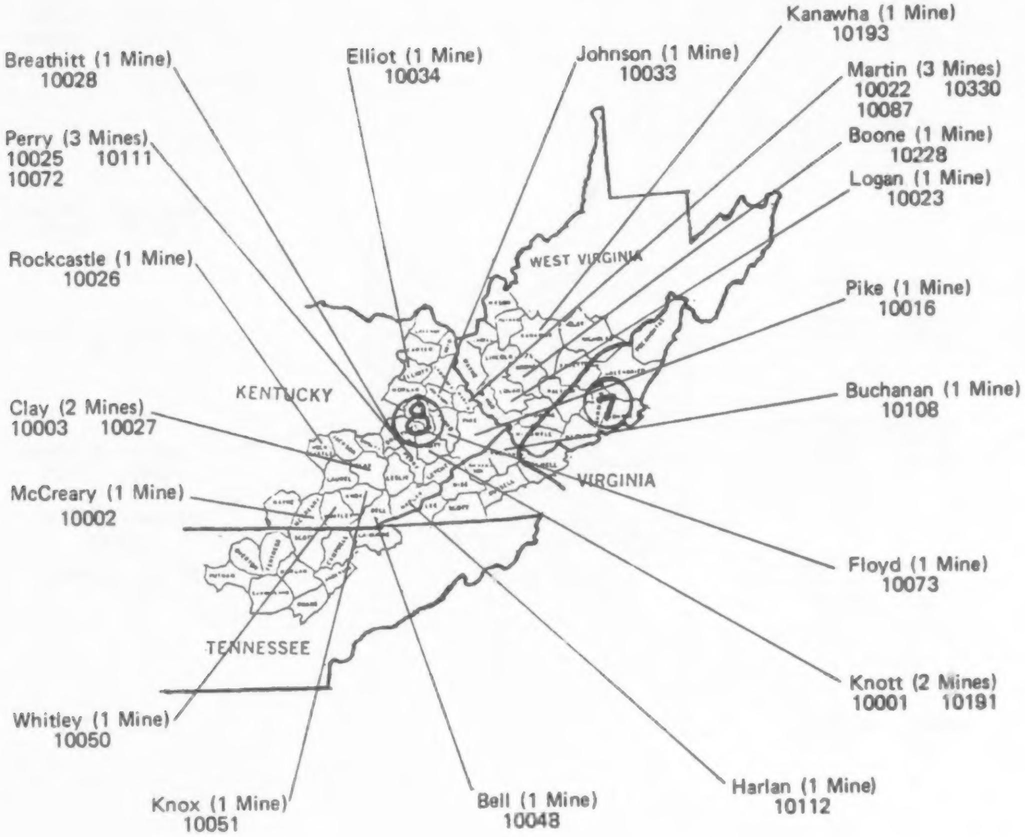
Morgan (1 Mine)
10164

Unknown County (18 Mines)
10132 10151 10157
10135 10152 10166
10136 10153 10168
10139 10154 10169
10140 10155 10170
10141 10156 10172

PRODUCING DISTRICTS 7 AND 8

VIRGINIA (District 7) 0 Mines
WEST VIRGINIA (District 7) 0 Mines

KENTUCKY (District 8) 21 Mines
TENNESSEE (District 8) 0 Mines
VIRGINIA (District 8) 1 Mine
WEST VIRGINIA (District 8) 3 Mines

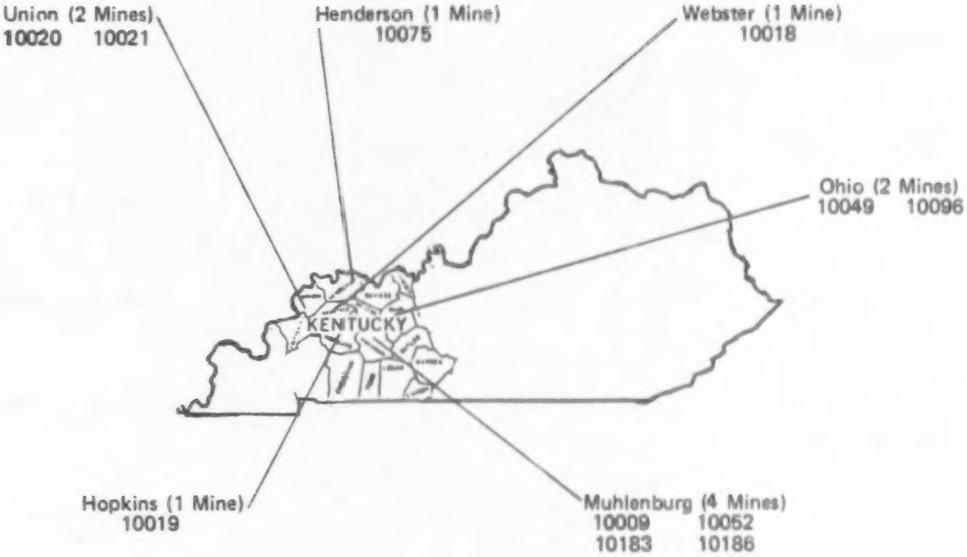

Breathitt (1 Mine) 10028
Elliot (1 Mine) 10034
Johnson (1 Mine) 10033
Kanawha (1 Mine) 10193
Martin (3 Mines) 10022 10330 10087
Boone (1 Mine) 10228
Logan (1 Mine) 10023
Perry (3 Mines) 10025 10111 10072
Rockcastle (1 Mine) 10026
WEST VIRGINIA
Pike (1 Mine) 10016
KENTUCKY
8
7
Buchanan (1 Mine) 10108
Clay (2 Mines) 10003 10027
VIRGINIA
McCreary (1 Mine) 10002
Floyd (1 Mine) 10073
TENNESSEE
Knott (2 Mines) 10001 10191
Whitley (1 Mine) 10050
Harlan (1 Mine) 10112
Knox (1 Mine) 10051
Bell (1 Mine) 10048

Unknown, Kentucky (1 Mine)
10110

Unknown, (District 8) (3 Mines)
10180 10182
10181

PRODUCING DISTRICT 9

WESTERN KENTUCKY 12 Mines



Union (2 Mines)
10020 10021

Henderson (1 Mine)
10075

Webster (1 Mine)
10018

Ohio (2 Mines)
10049 10096

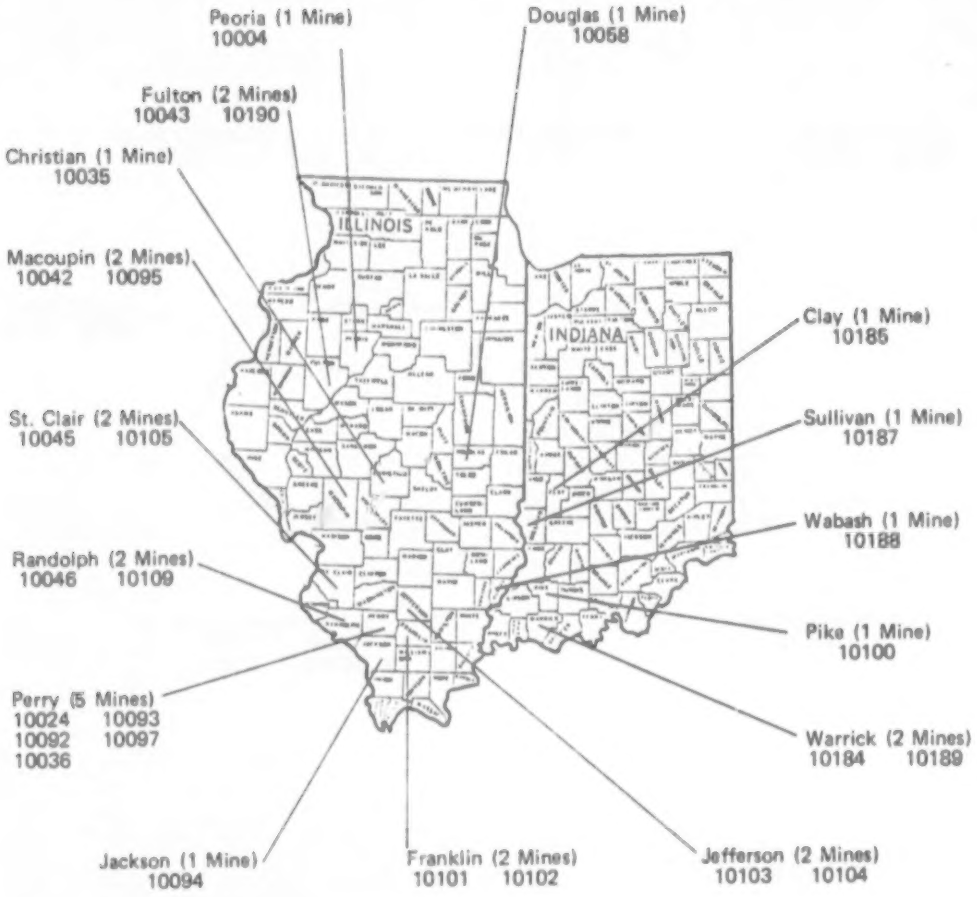
Hopkins (1 Mine)
10019

Muhlenburg (4 Mines)
10009 10052
10183 10186

Unknown County (1 Mine)
10056

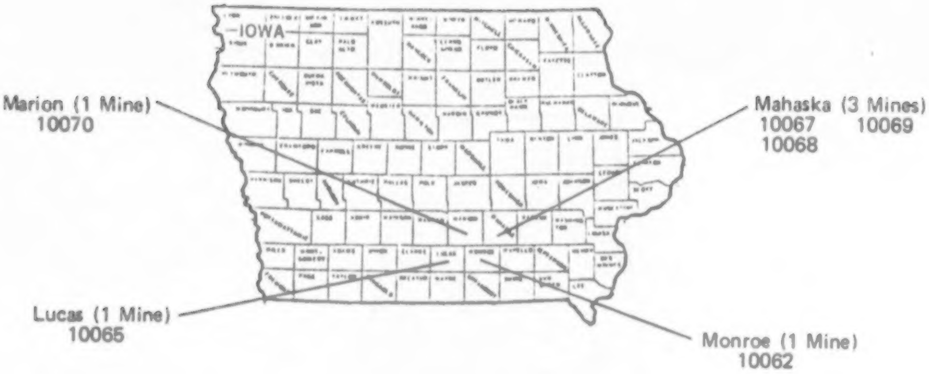
PRODUCING DISTRICTS 10 AND 11

ILLINOIS (District 10) 22 Mines
INDIANA (District 11) 5 Mines


Peoria (1 Mine) 10004
Douglas (1 Mine) 10058
Fulton (2 Mines) 10043 10190
Christian (1 Mine) 10035
Macoupin (2 Mines) 10042 10095
St. Clair (2 Mines) 10045 10105
Randolph (2 Mines) 10046 10109
Perry (5 Mines) 10024 10093 10092 10097 10036
Jackson (1 Mine) 10094
Franklin (2 Mines) 10101 10102
Jefferson (2 Mines) 10103 10104
Warrick (2 Mines) 10184 10189
Pike (1 Mine) 10100
Wabash (1 Mine) 10188
Sullivan (1 Mine) 10187
Clay (1 Mine) 10185
ILLINOIS
INDIANA

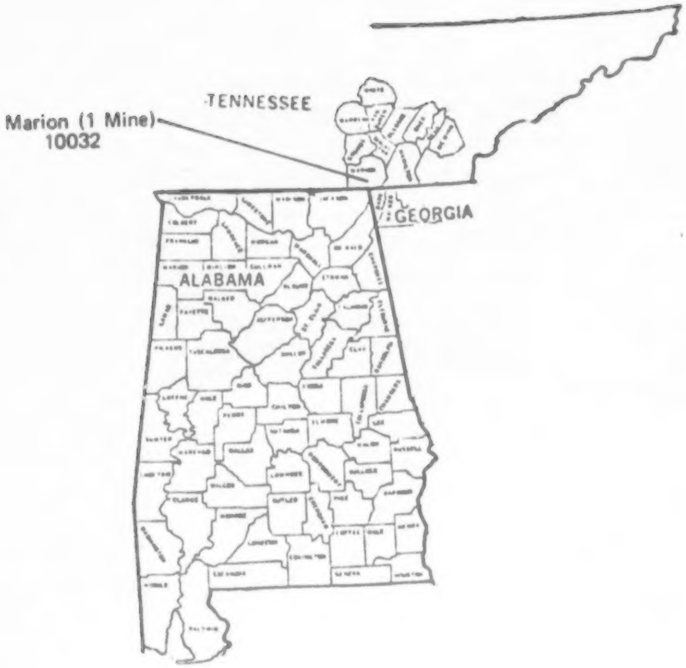
PRODUCING DISTRICT 12

IOWA 6 Mines


IOWA
Marion (1 Mine)
10070
Mahaska (3 Mines)
10067 10069
10068
Lucas (1 Mine)
10065
Monroe (1 Mine)
10062

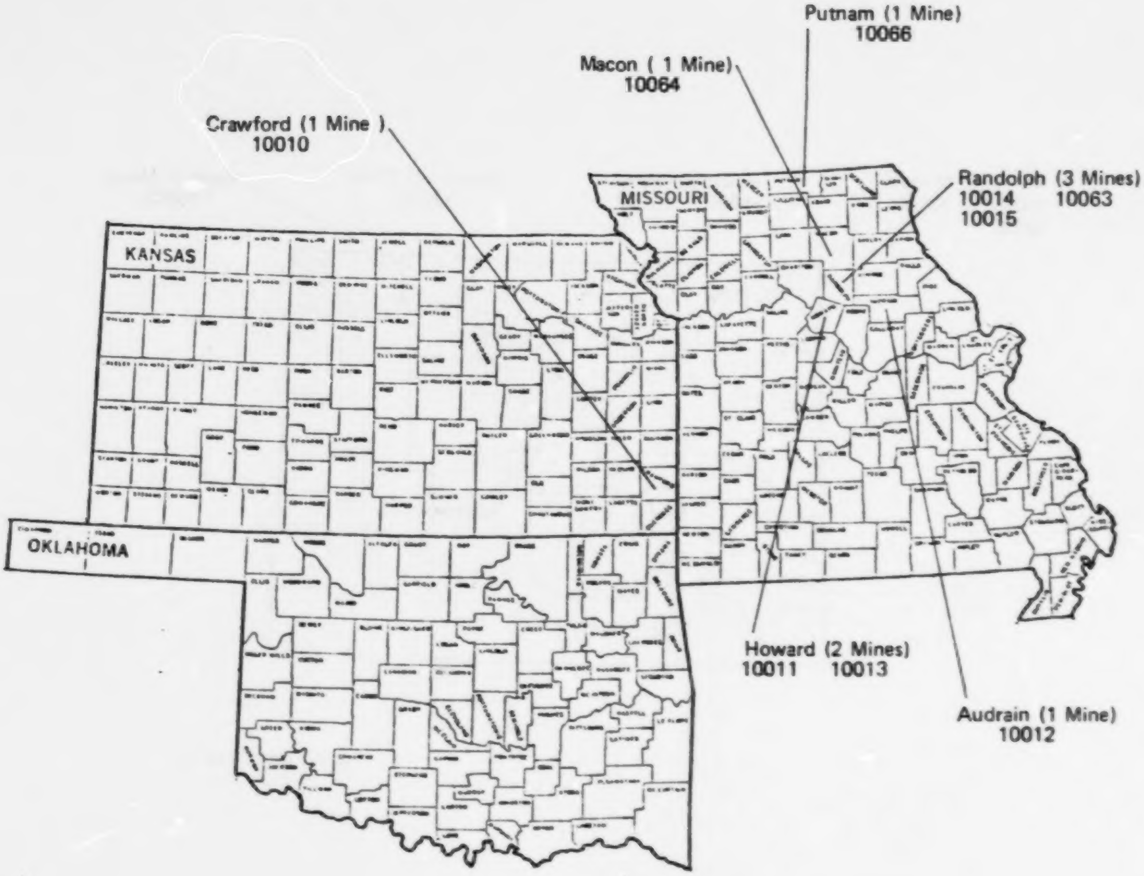
PRODUCING DISTRICT 13

ALABAMA 0 Mines
TENNESSEE 1 Mine


Marion (1 Mine)
10032
TENNESSEE
GEORGIA
ALABAMA

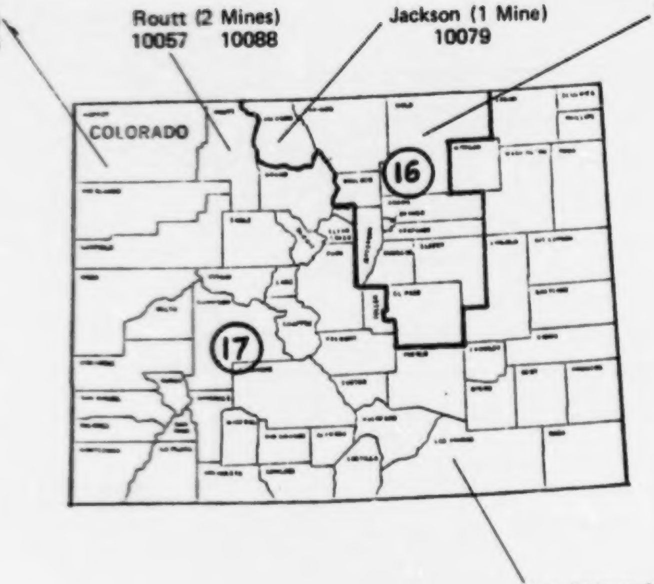
PRODUCING DISTRICT 15

KANSAS	1 Mine
MISSOURI	8 Mines
OKLAHOMA	0 Mines


Putnam (1 Mine)
10066
Macon (1 Mine)
10064
Crawford (1 Mine)
10010
Randolph (3 Mines)
10014 10063
10015
MISSOURI
KANSAS
OKLAHOMA
Howard (2 Mines)
10011 10013
Audrain (1 Mine)
10012

PRODUCING DISTRICTS 16 AND 17

COLORADO
District 16 2 Mines
District 17 6 Mines



Moffat (3 Mines)
10089 10106
10090

Routt (2 Mines)
10057 10088

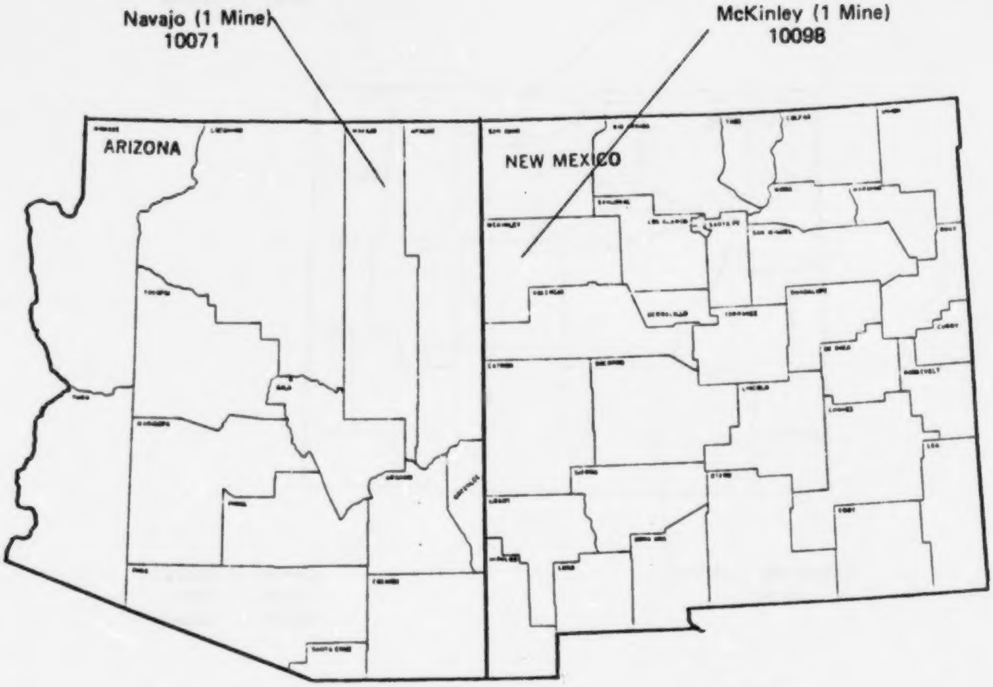
Jackson (1 Mine)
10079

Weld (1 Mine)
10078

Las Animas (1 Mine)
10091

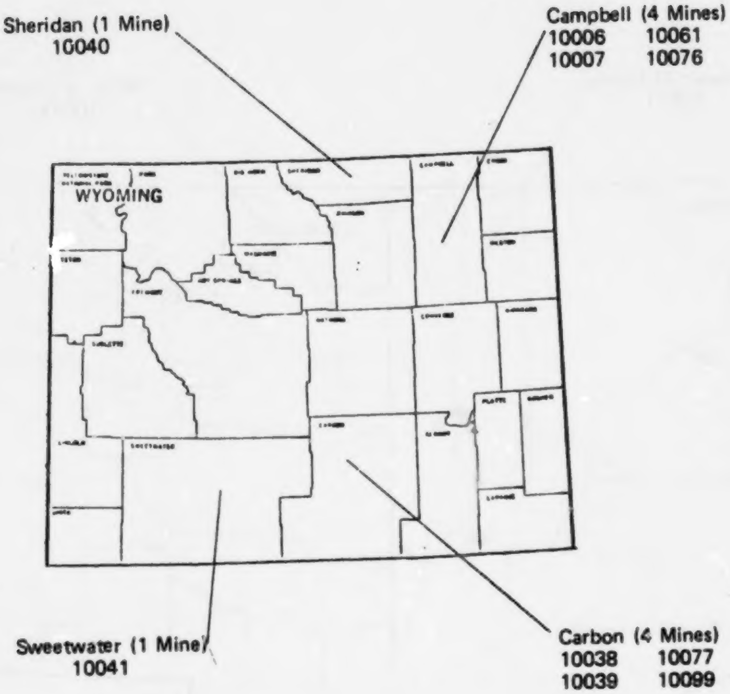
PRODUCING DISTRICT 18

ARIZONA 1 Mine
NEW MEXICO 1 Mine


Navajo (1 Mine)
10071
McKinley (1 Mine)
10098
ARIZONA
NEW MEXICO

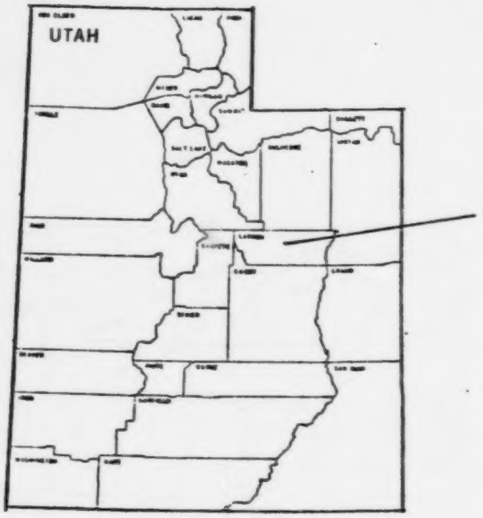
PRODUCING DISTRICT 19

WYOMING 10 Mines


Sheridan (1 Mine)
10040
Campbell (4 Mines)
10006 10061
10007 10076
WYOMING
Sweetwater (1 Mine)
10041
Carbon (4 Mines)
10038 10077
10039 10099

PRODUCING DISTRICT 20

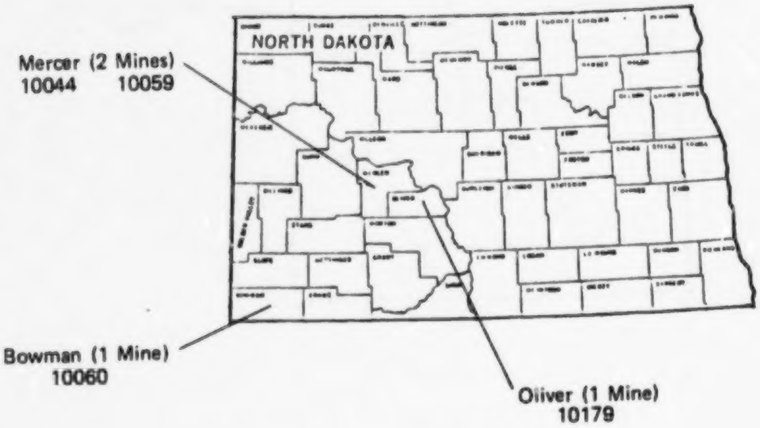
UTAH 1 Mine



Carbon (1 Mine)
10107

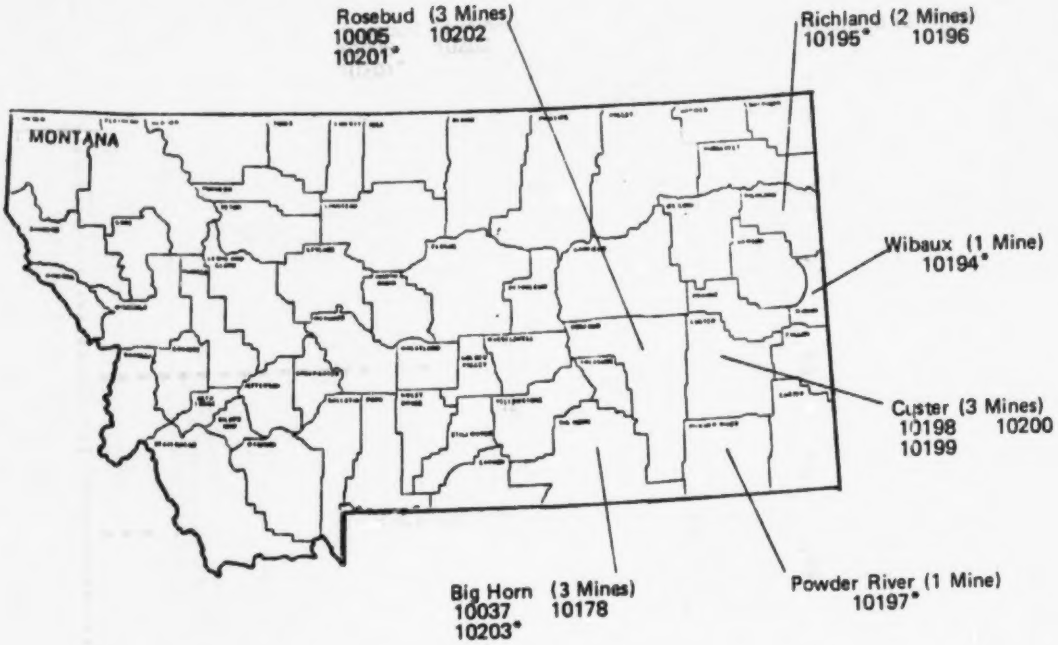
PRODUCING DISTRICT 21

NORTH DAKOTA 4 Mines


NORTH DAKOTA
Mercer (2 Mines)
10044 10059
Bowman (1 Mine)
10060
Oliver (1 Mine)
10179

PRODUCING DISTRICT 22

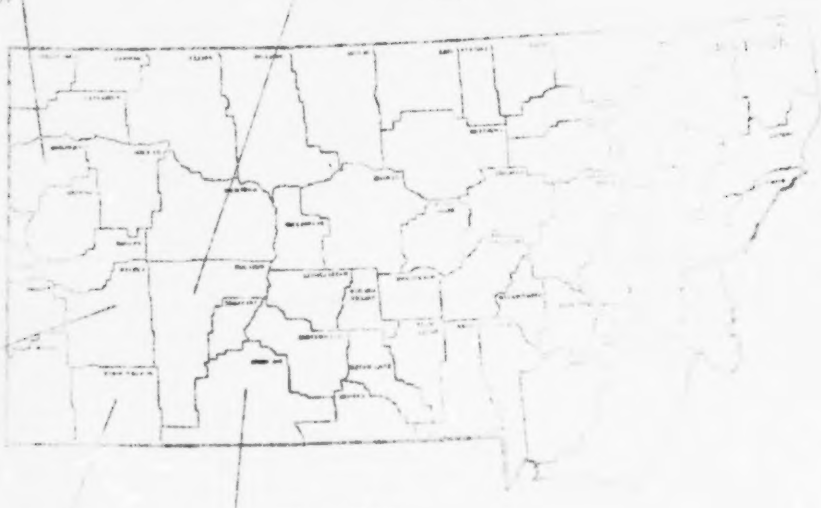
MONTANA 13 Mines


Rosebud (3 Mines)
10005 10202
10201*
Richland (2 Mines)
10195* 10196
MONTANA
Wibaux (1 Mine)
10194*
Custer (3 Mines)
10198 10200
10199
Big Horn (3 Mines)
10037 10178
10203*
Powder River (1 Mine)
10197*

* Core Samples, No production.

PRODUCING DISTRICT 22

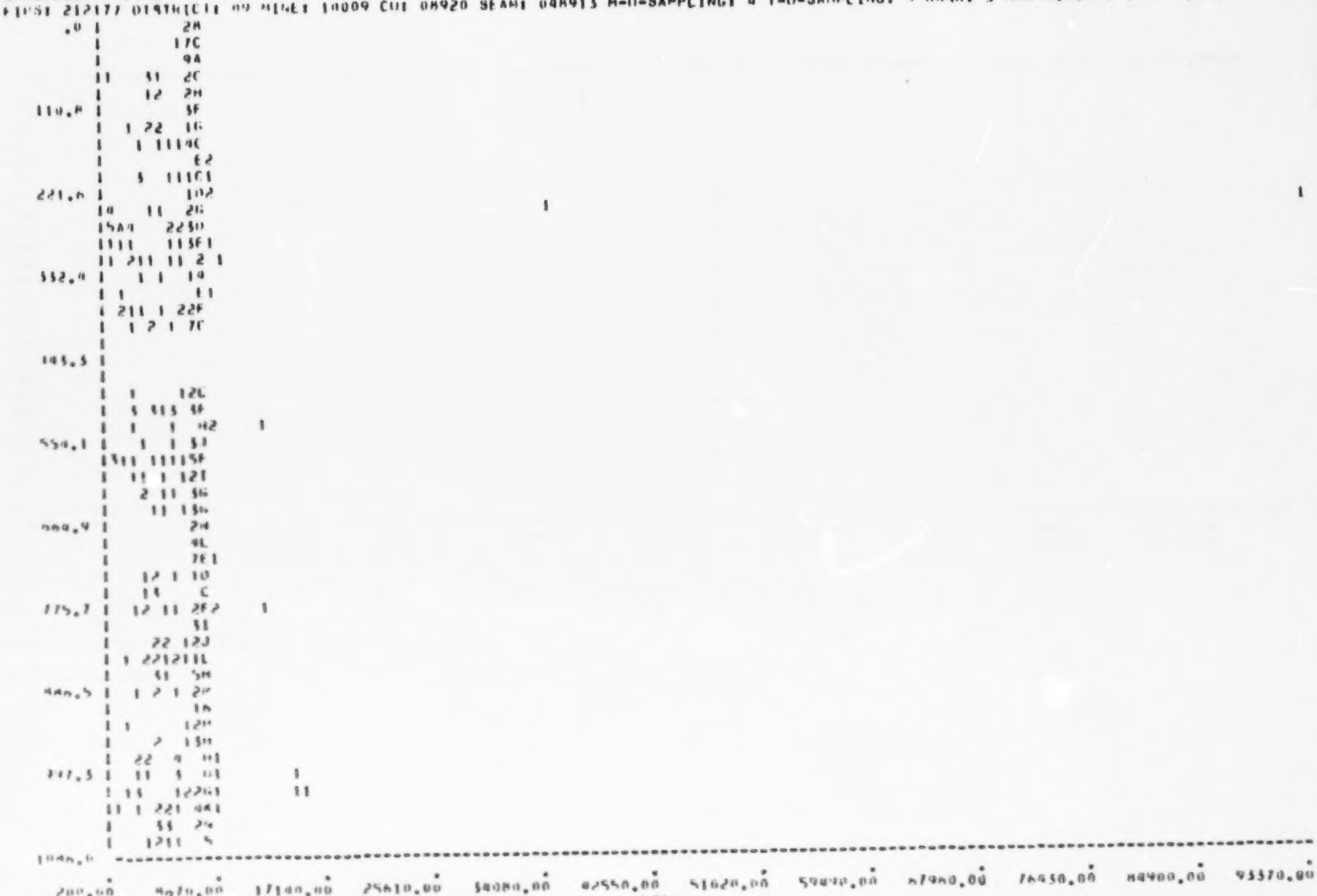
MONTANA 13 Mines


Richland (2 Mines)
10195* 10196
Rosebud (3 Mines)
10005 10202
10201*
Wibaux (1 Mine)
10194*
Carbon (3 Mines)
10198 [illegible]
10199
Powder River (1 Mine)
10197*
Big Horn (3 Mines)
10027 10178
10203*

* Core Samples, No production.

APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM

```
                                                                                                   MINING DATE IS 740109
                                                                                                         - N=  1027
ANALYSIS OF TONS
FIRST 212177 DISTRICT: 09 MINE: 10009 CU: 08920 SEAM: 048913 M-O-SAMPLING: 4 T-O-SAMPLING: 4 RANK: 3 M-O-MINING: 1 PREP.: 1
```



```
SAMPLE STATISTICS: MEAN, SD, WSD --- 6368.5569 5515.1966      .5209
EMPLOYEE: 09  08920  10009     252   13960.     3.82  10789.     6.72    1043
```

APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM

```
CHECK#21 09  08920  10009     252   92600.    3.67  11299.    6.17    1042

NORMAL DISTRIBUTION - OBSERVED

CELL #               1            2            3            4            5            6            7            8            9

FREQUENCY            0            0           34          120          864            2            4            1            2
FROM               .00     -5238.63     -1923.43      1391.76      4706.96      8022.16     11337.35     14652.55     17967.74
TO            -5238.63     -1923.43      1391.76      4706.96      8022.16     11337.35     14652.55     17967.74    *********
-----------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 35 POINTS

   840                                                                    *
   805                                                                    *
   770                                                                    *
   735                                                                    *
   700                                                                    *
   665                                                                    *
   630                                                                    *
   595                                                                    *
   560                                                                    *
   525                                                                    *
   490                                                                    *
   455                                                                    *
   420                                                                    *
   385                                                                    *
   350                                                                    *
   315                                                                    *
   280                                                                    *
   245                                                                    *
   210                                                                    *
   175                                                                    *
   140                                                                    *
   105                                                       *            *
    70                                                       *            *
    35                                                       *            *
-----------------------------------------------------------------------------------------------------------------------------
```

```
NORMAL DISTRIBUTION - EXPECTED

CELL #              1          2          3          4          5          6          7          8          9

FREQUENCY           0          6         62        248        393        248         62          6          0
FROM              .00   -5238.63   -1923.43    1391.76    4706.96    8022.16   11337.35   14652.55   17967.74
TO           -5238.63   -1923.43    1391.76    4706.96    8022.16   11337.35   14652.55   17967.74  *********
-------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

   384                                                          *
   368                                                          *
   352                                                          *
   336                                                          *
   320                                                          *
   304                                                          *
   288                                                          *
   272                                                          *
   256                                                          *
   240                                               *          *          *
   224                                               *          *          *
   208                                               *          *          *
   192                                               *          *          *
   176                                               *          *          *
   160                                               *          *          *
   144                                               *          *          *
   128                                               *          *          *
   112                                               *          *          *
    96                                               *          *          *
    80                                               *          *          *
    64                                               *          *          *
    48                                    *          *          *          *          *
    32                                    *          *          *          *          *
    16                                    *          *          *          *          *
-------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1           0         .2          .2
  2           0        6.1         5.6
  3          34       62.2        27.7
  4         120      248.3       127.8
  5         864      393.3       470.2
  6           2      248.3       245.8
  7           4       62.2        57.7
  8           1        6.1         4.6
  9           2         .2         1.5

CHI SQUAREDS ARE       952.933        945.903        937.219 FOR 6,4,2 DEGREES OF FREEDOM
```

APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM

```
INVERTED GAMMA DISTRIBUTION - EXPECTED

CELL #              1          2          3          4          5          6          7          8          9

FREQUENCY           0          0          0        341        470        148         43         14          5
FROM:             .00   -5238.63   -1923.43    1391.76    4706.96    8022.16   11337.35   14652.55   17967.74
TO:          -5238.63   -1923.43    1391.76    4706.96    8022.16   11337.35   14652.55   17967.74  *********
-----------------------------------------------------------------------------------------------------------------
EACH * EQUALS 19 POINTS

  456                                                           *
  437                                                           *
  418                                                           *
  399                                                           *
  380                                                           *
  361                                                           *
  342                                                           *
  323                                                *          *
  304                                                *          *
  285                                                *          *
  266                                                *          *
  247                                                *          *
  228                                                *          *
  209                                                *          *
  190                                                *          *
  171                                                *          *
  152                                                *          *
  133                                                *          *          *
  114                                                *          *          *
   95                                                *          *          *
   76                                                *          *          *
   57                                                *          *          *
   38                                                *          *          *          *
   19                                                *          *          *          *
-----------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1           0         .0          .0
  2           0         .0          .0
  3          34         .0        33.5
  4         120      341.3       220.8
  5         864      470.4       393.1
  6           2      147.9       145.4
  7           4       43.3        38.8
  8           1       14.3        12.8
  9           2        5.4         2.9

CHI SQUARES ARE     141891.699    141890.145    141878.664 FOR 6,4,2 DEGREES OF FREEDOM
SAMPLE STATISTICS: MEAN, SD, RSD ---      8.6627      .5248      .0606
```

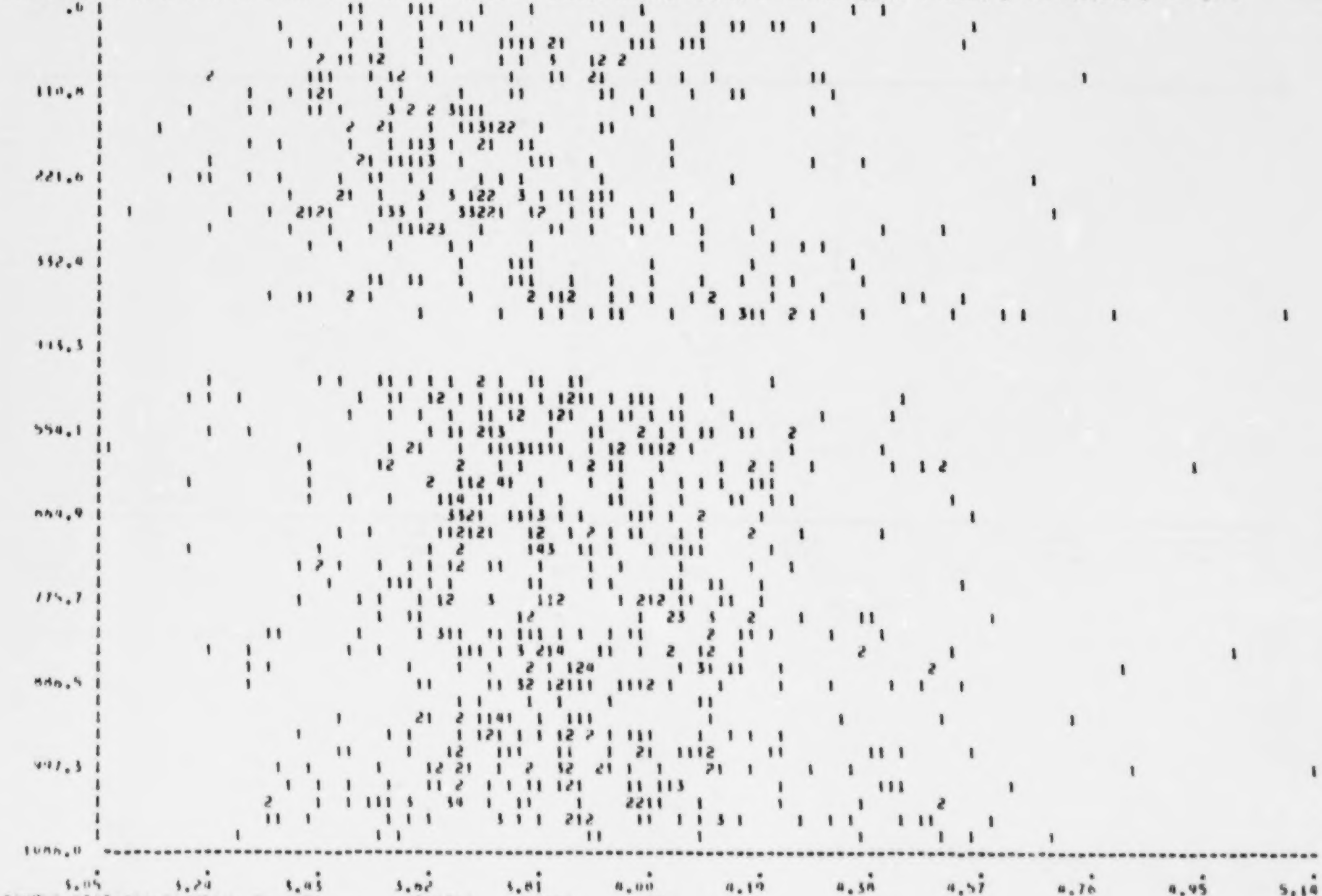
APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM

```
LOG NORMAL DISTRIBUTION - OBSERVED

CELL #              1         2         3         4         5         6         7         8         9

FREQUENCY          22        16        41        65       859        19         3         1         1
FROM:             .00      6.85      7.35      7.88      8.40      8.93      9.45      9.97     10.50
TO:              6.85      7.35      7.88      8.40      8.93      9.45      9.97     10.50 *********
---------------------------------------------------------------------------------------------------
EACH * EQUALS 35 POINTS

   840                                            *
   805                                            *
   770                                            *
   735                                            *
   700                                            *
   665                                            *
   630                                            *
   595                                            *
   560                                            *
   525                                            *
   490                                            *
   455                                            *
   420                                            *
   385                                            *
   350                                            *
   315                                            *
   280                                            *
   245                                            *
   210                                            *
   175                                            *
   140                                            *
   105                                            *
    70                                            *
    35                        *         *         *
---------------------------------------------------------------------------------------------------
```

APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM

```
LOG NORMAL DISTRIBUTION - EXPECTED

CELL #              1         2         3         4         5         6         7         8         9

FREQUENCY           0         6        62       288       393       288        62         6         0
FROM              .00      6.83      7.35      7.88      8.40      8.93      9.45      9.97     10.50
TO               6.83      7.35      7.88      8.40      8.93      9.45      9.97     10.50 *********
----------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

   384                                                      *
   368                                                      *
   352                                                      *
   336                                                      *
   320                                                      *
   304                                                      *
   288                                                      *
   272                                                      *
   256                                                      *
   240                                            *         *         *
   224                                            *         *         *
   208                                            *         *         *
   192                                            *         *         *
   176                                            *         *         *
   160                                            *         *         *
   144                                            *         *         *
   128                                            *         *         *
   112                                            *         *         *
    96                                            *         *         *
    80                                            *         *         *
    64                                            *         *         *
    48                                  *         *         *         *         *
    32                                  *         *         *         *         *
    16                                  *         *         *         *         *
----------------------------------------------------------------------------------------------------

CELL    OBSERVED   EXPECTED   DIFFERENCE
 1            22         .2         21.3
 2            16        6.1          9.4
 3            41       62.2         20.7
 4            65      288.3        182.8
 5           459      393.3        465.2
 6            19      288.3        228.8
 7             3       62.2         58.7
 8             1        6.1          4.6
 9             1         .2           .3

CHI SQUARES ARE      2667.878        475.827        958.045 FOR 6,4,2 DEGREES OF FREEDOM
```

```
ANALYSIS OF SULFUR                                                                                  = N=  1027
FIRST 212177 DISTRICTS BY MINE: 10009 CO: 08920 SEAM: 088913 M-D-SAMPLING: 4 T-D-SAMPLING: 4 RANK: 3 M-U-MINING: 1 PREP.: 1
```



```
SAMPLE STATISTICS: MEAN, SD, RSD ---     3.8203     .3091     .0796
```

```
NORMAL DISTRIBUTION - OBSERVED

CELL #             1         2         3         4         5         6         7         8         9

FREQUENCY          0         1        47       291       402       207        60        11         4
FROM             .00      2.76      3.06      3.36      3.67      3.97      4.28      4.58      4.88
TO              2.76      3.06      3.36      3.67      3.97      4.28      4.58      4.88 *********
----------------------------------------------------------------------------------------------------
EACH * EQUALS 17 POINTS

  391                                                      *
  374                                                      *
  357                                                      *
  340                                                      *
  323                                                      *
  306                                                      *
  289                                            *         *
  272                                            *         *
  255                                            *         *
  238                                            *         *
  221                                            *         *
  204                                            *         *         *
  187                                            *         *         *
  170                                            *         *         *
  153                                            *         *         *
  136                                            *         *         *
  119                                            *         *         *
  102                                            *         *         *
   85                                            *         *         *
   68                                            *         *         *
   51                                            *         *         *         *
   34                                  *         *         *         *         *
   17                                  *         *         *         *         *
----------------------------------------------------------------------------------------------------
```

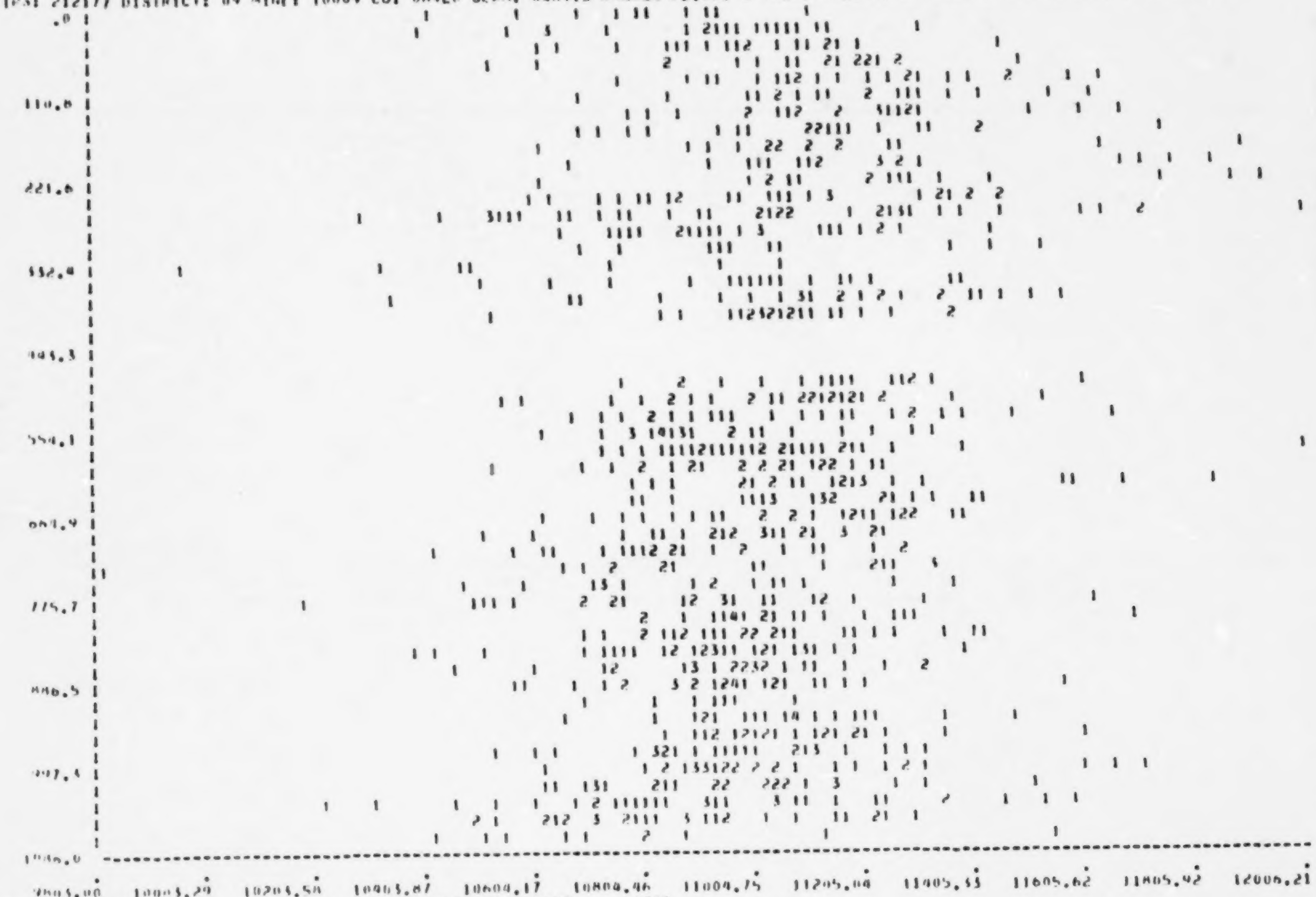
```
NORMAL DISTRIBUTION - EXPECTED

CELL #               1            2            3            4            5            6            7            8            9

FREQUENCY            0            6           62          248          393          248           62            6            0
FROM:              .00         2.76         3.06         3.36         3.67         3.97         4.28         4.58         4.98
TO:               2.76         3.06         3.36         3.67         3.97         4.28         4.58         4.88    *********
----------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

  384                                                                    *
  368                                                                    *
  352                                                                    *
  336                                                                    *
  320                                                                    *
  304                                                                    *
  288                                                                    *
  272                                                                    *
  256                                                                    *
  240                                                       *            *            *
  224                                                       *            *            *
  208                                                       *            *            *
  192                                                       *            *            *
  176                                                       *            *            *
  160                                                       *            *            *
  144                                                       *            *            *
  128                                                       *            *            *
  112                                                       *            *            *
   96                                                       *            *            *
   80                                                       *            *            *
   64                                                       *            *            *
   48                                          *            *            *            *            *
   32                                          *            *            *            *            *
   16                                          *            *            *            *            *
----------------------------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1           0         .2           .2
  2           1        6.1          4.6
  3          47       62.2         14.7
  4         291      248.3         42.2
  5         402      393.3          8.2
  6         207      248.3         40.8
  7          64       62.2          1.3
  8          11        6.1          4.4
  9           4         .2          3.3

CHI SQUARES ARE         69.670        24.170        17.566 FOR 6,4,2 DEGREES OF FREEDOM
```

```
INVERTED GAMMA DISTRIBUTION - EXPECTED

CELL #             1          2          3          4          5          6          7          8          9

FREQUENCY          0          2         56        272        395        226         68         10          1
FROM:            .00       2.76       3.06       3.36       3.67       3.97       4.28       4.58       4.88
TO:             2.76       3.06       3.36       3.67       3.97       4.28       4.58       4.88  *********
----------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

  384                                                      *
  368                                                      *
  352                                                      *
  336                                                      *
  320                                                      *
  304                                                      *
  288                                                      *
  272                                           *          *
  256                                           *          *
  240                                           *          *
  224                                           *          *          *
  208                                           *          *          *
  192                                           *          *          *
  176                                           *          *          *
  160                                           *          *          *
  144                                           *          *          *
  128                                           *          *          *
  112                                           *          *          *
   96                                           *          *          *
   80                                           *          *          *
   64                                           *          *          *          *
   48                                *          *          *          *          *
   32                                *          *          *          *          *
   16                                *          *          *          *          *
----------------------------------------------------------------------------------------------------------------

CELL    OBSERVED    EXPECTED    DIFFERENCE
  1            0          .0            .0
  2            1         2.2            .7
  3           47        55.6           8.1
  4          291       272.5          18.0
  5          402       394.9           6.6
  6          207       226.3          18.8
  7           64        65.8           -.2
  8           11        10.5            .0
  9            4         1.1           2.4

CHI SQUAREDS ARE           9.109          4.278          4.053 FOR 6,4,2 DEGREES OF FREEDOM
SAMPLE STATISTICS: MEAN, SD, RSD ---      1.3372         .0783         .0586
```

```
LOG NORMAL DISTRIBUTION - OBSERVED

CELL #             1         2         3         4         5         6         7         8         9

FREQUENCY          0         2        53       284       389       222        65        10         2
FROM             .00      1.06      1.14      1.22      1.30      1.38      1.45      1.53      1.61
TO              1.06      1.14      1.22      1.30      1.38      1.45      1.53      1.61 *********
-----------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

 384                                                 *
 368                                                 *
 352                                                 *
 336                                                 *
 320                                                 *
 304                                                 *
 288                                                 *
 272                                       *         *
 256                                       *         *
 240                                       *         *
 224                                       *         *
 208                                       *         *         *
 192                                       *         *         *
 176                                       *         *         *
 160                                       *         *         *
 144                                       *         *         *
 128                                       *         *         *
 112                                       *         *         *
  96                                       *         *         *
  80                                       *         *         *
  64                                       *         *         *         *
  48                             *         *         *         *         *
  32                             *         *         *         *         *
  16                             *         *         *         *         *
-----------------------------------------------------------------------------------------------------
```

```
LOG NORMAL DISTRIBUTION - EXPECTED

CELL #               1            2            3            4            5            6            7            8            9

FREQUENCY            0            6           62          248          393          248           62            6            0
FROM              .00         1.06         1.14         1.22         1.30         1.38         1.45         1.53         1.61
TO               1.06         1.14         1.22         1.30         1.38         1.45         1.53         1.61    *********
------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS
                                                                         *
  384                                                                    *
  368                                                                    *
  352                                                                    *
  336                                                                    *
  320                                                                    *
  304                                                                    *
  288                                                                    *
  272                                                                    *
  256                                                       *            *            *
  240                                                       *            *            *
  224                                                       *            *            *
  192                                                       *            *            *
  176                                                       *            *            *
  160                                                       *            *            *
  144                                                       *            *            *
  128                                                       *            *            *
  112                                                       *            *            *
   96                                                       *            *            *
   80                                                       *            *            *
   64                                                       *            *            *
   48                                          *            *            *            *            *
   32                                          *            *            *            *            *
   16                                          *            *            *            *            *
------------------------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1           0         .2           .2
  2           2        6.1          3.6
  3          53       62.2          8.7
  4         284      248.5         35.2
  5         389      393.3          3.8
  6         222      248.3         25.8
  7          65       62.2          2.3
  8          10        6.1          3.4
  9           2         .2          1.3

CHI SQUARES ARE         20.047        13.017         9.020 FOR 6,4,2 DEGREES OF FREEDOM
```

```
                                                                                    - N=  1027
ANALYSIS OF HTH
FIRST 212177 DISTRICT: 09 MINE: 10009 CU: 08920 SEAM: 088915 M-D-SAMPLING: 6 T-D-SAMPLING: 8 RANK: 3 M-D-MINING: 1 PREP.: 1
```



```
SAMPLE STATISTICS: MEAN, SD, RSD --- 11007.4524  260.7489     .0241
```

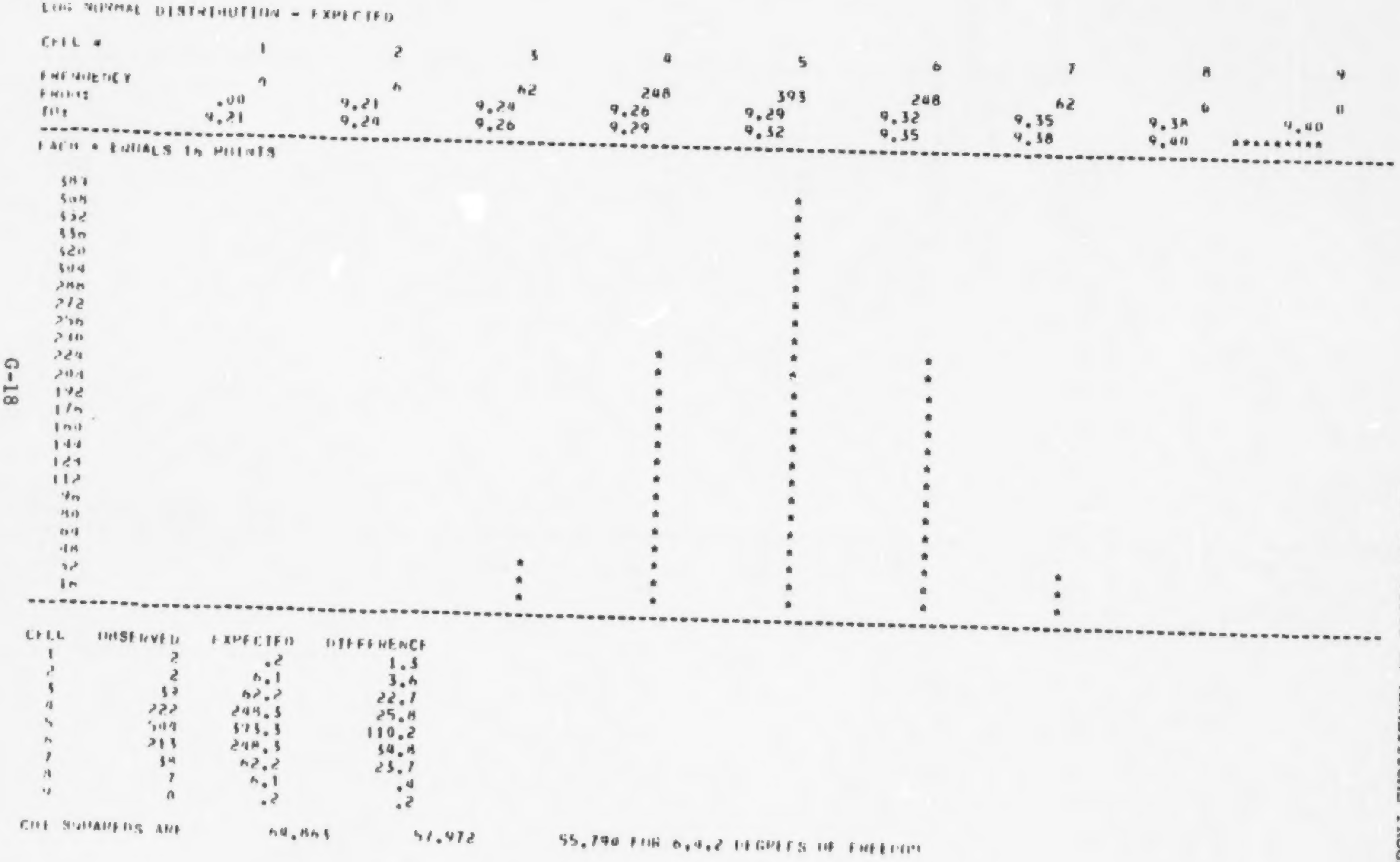
```
INITIAL DISTRIBUTION - OBSERVED

CELL #                   1              2              3              4              5              6              7              8              9

FREQUENCY                2              6             59            231            436            233             85             13              2
FROM                  .00       10080.83       10345.58       10610.33       10875.08       11139.83       11404.58       11669.33       11934.07
TO               10080.83       10345.58       10610.33       10875.08       11139.83       11404.58       11669.33       11934.07     **********
------------------------------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 18 POINTS

   432                                                                               *
   414                                                                               *
   396                                                                               *
   378                                                                               *
   360                                                                               *
   342                                                                               *
   324                                                                               *
   306                                                                               *
   288                                                                               *
   270                                                                               *
   252                                                                               *
   234                                                                               *
   216                                                                *              *              *
   198                                                                *              *              *
   180                                                                *              *              *
   162                                                                *              *              *
   144                                                                *              *              *
   126                                                                *              *              *
   108                                                                *              *              *
    90                                                                *              *              *
    72                                                                *              *              *
    54                                                 *              *              *              *
    36                                                 *              *              *              *              *
    18                                                 *              *              *              *              *
------------------------------------------------------------------------------------------------------------------------------------------------------
```

```
NORMAL DISTRIBUTION - EXPECTED

CELL #                1            2            3            4            5            6            7            8            9

FREQUENCY             0            6           62          248          393          248           62            6            0
FROM              .00     10080.83     10345.58     10610.33     10875.08     11139.83     11404.58     11669.33     11934.07
TO           10080.83     10345.58     10610.33     10875.08     11139.83     11404.58     11669.33     11934.07   **********
------------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

  384                                                                     *
  368                                                                     *
  352                                                                     *
  336                                                                     *
  320                                                                     *
  304                                                                     *
  288                                                                     *
  272                                                                     *
  256                                                                     *
  240                                                        *            *            *
  224                                                        *            *            *
  208                                                        *            *            *
  192                                                        *            *            *
  176                                                        *            *            *
  160                                                        *            *            *
  144                                                        *            *            *
  128                                                        *            *            *
  112                                                        *            *            *
   96                                                        *            *            *
   80                                                        *            *            *
   64                                                        *            *            *
   48                                           *            *            *            *            *
   32                                           *            *            *            *            *
   16                                           *            *            *            *            *
------------------------------------------------------------------------------------------------------------------------------------
CELL   OBSERVED   EXPECTED   DIFFERENCE
   1          2         .2          1.3
   2          6        6.1           .1
   3         59       62.2          2.7
   4        231      248.3         16.8
   5        436      393.3         42.2
   6        233      248.3         14.8
   7         95       62.2         16.7
   8         13        6.1          6.9
   9          2         .2          1.3

CHI SQUARED ARE          31.206        17.760        11.144 FOR 6,4,2 DEGREES OF FREEDOM
```

```
INVERTED GAMMA DISTRIBUTION - EXPECTED

CELL #               1            2            3            4            5            6            7            8            9

FREQUENCY            0            5           61          255          393          241           63            7            0
FROM             .00     10080.83     10345.58     10610.33     10875.08     11139.83     11404.58     11669.33     11934.07
TO          10080.83     10345.58     10610.33     10875.08     11139.83     11404.58     11669.33     11934.07   **********
--------------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

  384                                                                    *
  368                                                                    *
  352                                                                    *
  336                                                                    *
  320                                                                    *
  304                                                                    *
  288                                                                    *
  272                                                                    *
  256                                                                    *
  240                                                       *            *            *
  224                                                       *            *            *
  208                                                       *            *            *
  192                                                       *            *            *
  176                                                       *            *            *
  160                                                       *            *            *
  144                                                       *            *            *
  128                                                       *            *            *
  112                                                       *            *            *
   96                                                       *            *            *
   80                                                       *            *            *
   64                                                       *            *            *
   48                                          *            *            *            *            *
   32                                          *            *            *            *            *
   16                                          *            *            *            *            *
--------------------------------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED    DIFFERENCE
  1           2         .1           1.4
  2           6        4.8            .7
  3          59       60.8           1.3
  4         231      255.9          23.9
  5         436      393.4          42.1
  6         233      241.3           7.8
  7          45       63.1          17.6
  8          13        7.5           5.0
  9           2         .9           1.1

CHI SQUAREDS ARE        35.179        15.381        11.944 FOR 6,4,2 DEGREES OF FREEDOM
SAMPLE STATISTICS: MEAN, SD, RSD ---      9.3060      .0280      .0030
```

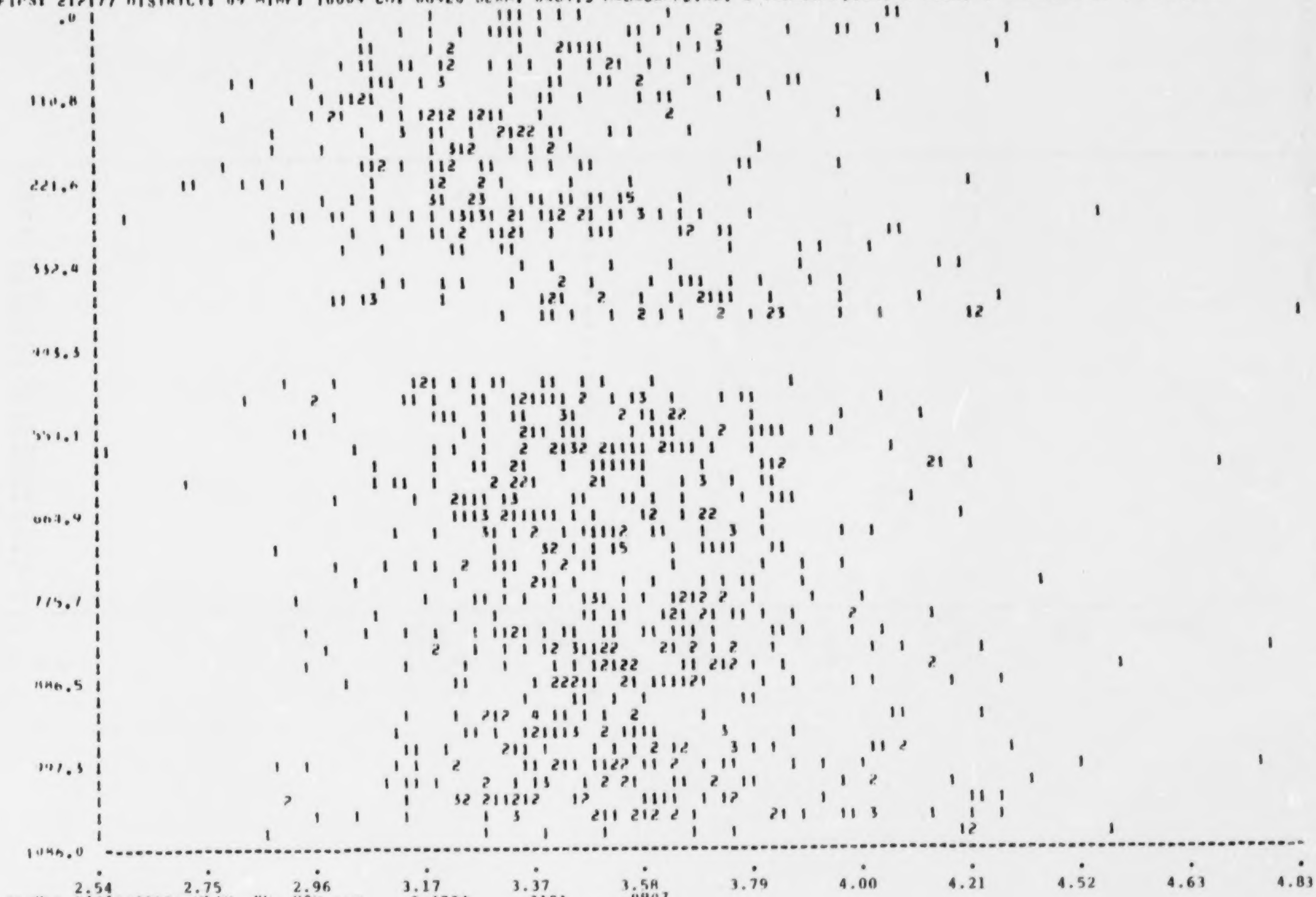
```
LOG NORMAL DISTRIBUTION - OBSERVED

CELL #                1            2            3            4            5            6            7            8            9

FREQUENCY             2            2           39          222          508          213           38            7            0
FROM               .00         9.21         9.24         9.26         9.29         9.32         9.35         9.38         9.40
TO:               9.21         9.24         9.26         9.29         9.32         9.35         9.38         9.40    *********
-------------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 21 POINTS

  504                                                                     *
  483                                                                     *
  462                                                                     *
  441                                                                     *
  420                                                                     *
  399                                                                     *
  378                                                                     *
  357                                                                     *
  336                                                                     *
  315                                                                     *
  294                                                                     *
  273                                                                     *
  252                                                                     *
  231                                                                     *
  210                                                        *            *            *
  189                                                        *            *            *
  168                                                        *            *            *
  147                                                        *            *            *
  126                                                        *            *            *
  105                                                        *            *            *
   84                                                        *            *            *
   63                                                        *            *            *
   42                                                        *            *            *
   21                                           *            *            *            *            *
-------------------------------------------------------------------------------------------------------------------------------------
```

APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM



```
LOG NORMAL DISTRIBUTION - EXPECTED

CELL #              1           2           3           4           5           6           7           8           9

FREQUENCY           0           6          62         248         393         248          62           6           0
FROM              .00        9.21        9.24        9.26        9.29        9.32        9.35        9.38        9.40
TO               9.21        9.24        9.26        9.29        9.32        9.35        9.38        9.40   *********
---------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

  384
  368
  352
  336
  320
  304
  288
  272
  256
  240
  224
  208
  192
  176
  160
  144
  128
  112
   96
   80
   64
   48
   32
   16
---------------------------------------------------------------------------------------------------------------------
CELL   OBSERVED   EXPECTED   DIFFERENCE
  1          2         .2          1.5
  2          2        6.1          3.6
  3         37       62.2         22.1
  4        222      248.5         25.8
  5        504      393.3        110.2
  6        213      248.3         34.8
  7         34       62.2         23.7
  8          7        6.1           .4
  9          0         .2           .2

CHI SQUARES ARE            68.865         57.972         55.790 FOR 6,4,2 DEGREES OF FREEDOM
```

```
ANALYSIS OF LBS SULFUR/MMBtu                                                                         = N=  1627
FIRST 212177 DISTRICT: 09 MINE: 10009 CO: 08920 SEAM: 088913 M-O-SAMPLING: 8 T-O-SAMPLING: 8 RANK: 5 M-O-MINING: 1 PREP.: 1
```



```
SAMPLE STATISTICS: MEAN, SD, RSD ---     3.4724      .3151      .0907
```

```
NORMAL DISTRIBUTION - OBSERVED

CELL #                1        2        3        4        5        6        7        8        9

FREQUENCY             0        5       49      264      420      205       70        8        4
FROM                .00     4.50     5.10     5.70     6.30     6.90     7.50     8.09     8.69
TO                 4.50     5.10     5.70     6.30     6.90     7.50     8.09     8.69*********
-----------------------------------------------------------------------------------------------
EACH * EQUALS 17 POINTS

   408                                            *
   391                                            *
   374                                            *
   357                                            *
   340                                            *
   323                                            *
   306                                            *
   289                                            *
   272                                            *
   255                                   *        *
   238                                   *        *
   221 .                                 *        *
   204                                   *        *        *
   187                                   *        *        *
   170                                   *        *        *
   153                                   *        *        *
   136                                   *        *        *
   119                                   *        *        *
   102                                   *        *        *
    85                                   *        *        *
    68                                   *        *        *        *
    51                                   *        *        *        *
    34                          *        *        *        *        *
    17                          *        *        *        *        *
-----------------------------------------------------------------------------------------------
```

```
NORMAL DISTRIBUTION - EXPECTED

CELL #               1            2            3            4            5            6            7            8            9

FREQUENCY            0            6           62          248          393          248           62            6            0
FROM             .00         4.50         5.10         5.70         6.30         6.90         7.50         8.09         8.69
TO              4.50         5.10         5.70         6.30         6.90         7.50         8.09         8.69    *********
----------------------------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

   384                                                                   *
   368                                                                   *
   352                                                                   *
   336                                                                   *
   320                                                                   *
   304                                                                   *
   288                                                                   *
   272                                                                   *
   256                                                                   *
   240                                                      *            *            *
   224                                                      *            *            *
   208                                                      *            *            *
   192                                                      *            *            *
   176                                                      *            *            *
   160                                                      *            *            *
   144                                                      *            *            *
   128                                                      *            *            *
   112                                                      *            *            *
    96                                                      *            *            *
    80                                                      *            *            *
    64                                                      *            *            *
    48                                         *            *            *            *            *
    32                                         *            *            *            *            *
    16                                         *            *            *            *            *
----------------------------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1           0         .2           .2
  2           3        6.1          2.6
  3          49       62.2         12.7
  4         264      248.3         15.2
  5         420      393.3         30.2
  6         205      248.3         42.8
  7          70       62.2          7.3
  8           8        6.1          1.4
  9           0         .2          3.3

CHI SQUARES ARE          81.015          15.510          14.078 FOR 6,4,2 DEGREES OF FREEDOM
```

```
INVERTED GAMMA DISTRIBUTION - EXPECTED

CELL #             1         2         3         4         5         6         7         8         9

FREQUENCY          0         2        55       276       399       223        64        11         1
FROM             .00      4.50      5.10      5.70      6.30      6.90      7.50      8.09      8.69
TO              4.50      5.10      5.70      6.30      6.90      7.50      8.09      8.69 *********
----------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

   384                                                     *
   368                                                     *
   352                                                     *
   336                                                     *
   320                                                     *
   304                                                     *
   288                                                     *
   272                                           *         *
   256                                           *         *
   240                                           *         *
   224                                           *         *
   208                                           *         *         *
   192                                           *         *         *
   176                                           *         *         *
   160                                           *         *         *
   144                                           *         *         *
   128                                           *         *         *
   112                                           *         *         *
    96                                           *         *         *
    80                                           *         *         *
    64                                           *         *         *         *
    48                                 *         *         *         *         *
    32                                 *         *         *         *         *
    16                                 *         *         *         *         *
----------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1          0         .0           .0
  2          3        1.8           .7
  3         49       54.7          5.2
  4        264      276.0         11.5
  5        428      399.5         29.0
  6        205      223.3         17.8
  7         70       64.0          5.5
  8          8       11.2          2.7
  9          4        1.9          2.1

CHI SQUAREDS ARE          9.239         5.866         4.993 FOR 6,4,2 DEGREES OF FREEDOM
SAMPLE STATISTICS: MEAN, SD, RSD ---      1.8827      .0897      .0477
```

```
LOG NORMAL DISTRIBUTION - OBSERVED

CELL #              1         2         3         4         5         6         7         8         9

FREQUENCY           0         6        56       248       414       226        69         6         2
FROM:            .00      1.57      1.66      1.75      1.84      1.93      2.02      2.11      2.20
TO:             1.57      1.66      1.75      1.84      1.93      2.02      2.11      2.20 *********
-----------------------------------------------------------------------------------------------------
EACH * EQUALS 17 POINTS

   408                                                      *
   391                                                      *
   374                                                      *
   357                                                      *
   340                                                      *
   323                                                      *
   306                                                      *
   289                                                      *
   272                                                      *
   255                                                      *
   238                                            *         *
   221                                            *         *         *
   204                                            *         *         *
   187                                            *         *         *
   170                                            *         *         *
   153                                            *         *         *
   136                                            *         *         *
   119                                            *         *         *
   102                                            *         *         *
    85                                            *         *         *
    68                                            *         *         *         *
    51                                  *         *         *         *         *
    34                                  *         *         *         *         *
    17                                  *         *         *         *         *
-----------------------------------------------------------------------------------------------------
```

APPENDIX G:
SAMPLE OUTPUT OF ANALYTICAL PROGRAM

```
LOG NORMAL DISTRIBUTION - EXPECTED

CELL #                       1         2         3         4         5         6         7         8         9

FREQUENCY                    0         6        62       248       393       248        62         6         0
FROM                       .00      1.57      1.66      1.75      1.84      1.93      2.02      2.11      2.20
TO                        1.57      1.66      1.75      1.84      1.93      2.02      2.11      2.20 *********
----------------------------------------------------------------------------------------------------------------
EACH * EQUALS 16 POINTS

  384                                                               *
  368                                                               *
  352                                                               *
  336                                                               *
  320                                                               *
  304                                                               *
  288                                                               *
  272                                                               *
  256                                                               *
  240                                                     *         *         *
  224                                                     *         *         *
  208                                                     *         *         *
  192                                                     *         *         *
  176                                                     *         *         *
  160                                                     *         *         *
  144                                                     *         *         *
  128                                                     *         *         *
  112                                                     *         *         *
   96                                                     *         *         *
   80                                                     *         *         *
   64                                                     *         *         *
   48                                           *         *         *         *         *
   32                                           *         *         *         *         *
   16                                           *         *         *         *         *
----------------------------------------------------------------------------------------------------------------

CELL   OBSERVED   EXPECTED   DIFFERENCE
  1          0         .2          .2
  2          6        6.1          .1
  3         56       62.2         5.7
  4        244      248.3          .3
  5        418      393.3        20.2
  6        226      248.3        21.8
  7         69       62.2         6.3
  8          6        6.1          .1
  9          2         .2         1.3

CHI SQUARES ARE       11.198         9.113          4.107 FOR 6,4,2 DEGREES OF FREEDOM
```

SOURCE: FOSTER ASSOCIATES, INC.

APPENDIX H

SUMMARY OF SALIENT CHARACTERISTICS OF DATA SETS ANALYZED
(Statistical analysis based on data as received from respondents, disregarding lot-size)

Sheet 1 of 5

Mine Code (1)	FIPS Code (2)	USBM Dist. (3)	Red Code (4)	Rank (5)	Mining Method (6)	Preparation Code (7)	Method of Sampling (8)	Type of Sampling (9)	AVERAGE CHARACTERISTICS Volume (Tons) (10)	Sulfur Content (% A.R.) (11)	Heat Content (Btu/lb A.R.) (12)	Lbs Sulfur/ MMBtu (13)	RSD of Lbs Sulfur/ MMBtu (14)
10001	211119	8	010401	3	9	1	9	9	5318	2.69	11650	2.31	21.55
10002	211109	8	015109	3	9	2	9	9	3150	1.34	12423	1.08	23.76
10003	211051	8	100001	3	9	1	9	9	5688	2.18	11440	1.91	23.92
10004	170143	10	090002	3	2	2	4	4	9110	2.24	10588	2.11	18.49
10005 b/	300087	22	080800	2	2	1	1	4	8423	0.74	8639	.85	30.07
10005 b/	300087	22	080800	2	2	1	9	1	--	---	---	---	---
10005	300087	22	080800	2	2	1	9	2	--	---	---	---	---
10006	560005	19	095100	2	2	1	1	2	10147	0.36	8400	.42	12.36
10006	560005	19	095100	2	2	1	1	3	10513	0.36	8404	.43	11.94
10006	560005	19	095100	2	2	1	2	1	--	0.41	8376	.48	16.89
10006	560005	19	095100	2	2	1	1	2	11035	0.39	8308	.46	14.17
10006	560005	19	095100	2	2	1	1	3	10798	0.37	8360	.44	14.36
10006	560005	19	095100	2	2	1	1	3	10402	0.36	8389	.43	11.55
10007	560005	19	100003	2	2	1	1	2	10800	0.40	8298	.47	9.90
10007	560005	19	100003	2	2	1	1	3	10625	0.38	8406	.45	6.51
10008	560005	19	100002	2	2	1	9	5	11692	0.41	8440	.48	6.86
10009	212177	9	048913	3	1	1	1,4	4	6254	3.82	11008	3.47	9.08
10010	290037	15	049202	3	2	1,2	4	4	2496	2.99	10905	2.75	12.19
10011	290089	15	049202	3	2	1,2	4	4	706	3.87	10056	3.89	16.68
10012	290007	15	049008	3	2	2	4	4	505	3.77	10834	3.48	12.23
10013	290089	15	049202	3	2	2	4	4	712	3.68	10752	3.42	5.98
10014	290175	15	049202	3	2	1	4	4	904	4.03	9615	4.20	14.37
10015	290175	15	098700	3	2	1	4	4	580	4.74	9829	4.85	14.46
10016	211195	8	100004	3	1	2	9	4	10024	1.48	11836	1.25	10.52
10017	240001	1	001000	3	2	1	1,3,9	4	2055	1.61	11078	1.45	32.65
10018	212233	9	048415	3	1	1	9	4	4548	4.19	11054	3.80	5.67
10019	212107	9	048415	3	1	1	9	4	3739	3.65	10760	3.39	5.88
10019	212107	9	048415	3	1	2	1,4	4	5643	3.03	11197	2.70	13.43
10020	212226	9	048415	3	1	3	9	4	4031	2.99	10965	2.73	5.96
10021	212225	9	048415	3	1	3	9	4	4354	3.46	10577	3.27	5.30
10022	211159	8	100005	3	4	2	9	2	5060	0.61	12301	.49	4.94
10023	540045	8	015104	3	1	2	9	4	2594	0.73	11329	.64	8.89
10024	170157	10	090002	3	1	2	1	4	1470	3.49	10990	3.17	11.20
10025	211193	8	100006	3	9	1	4	4	1800	0.85	12404	.69	20.76
10026	211203	8	015108	3	9	1	4	4	1100	1.13	12195	.94	62.34
10027	211051	8	021202	3	9	1	4	4	1600	0.98	12936	.76	45.19
10028	211025	8	011108	3	9	1	4	4	300	0.68	12620	.54	14.36
10029	540061	3	003604	3	1	1	1	2	11870	2.80 d/	13054 d/	2.14	8.30
10030	390067	4	007402	3	1	1	1	2	10360	2.60 d/	12481 d/	2.09	6.56
10031	540023	1	007102	3	1	3	1	4	2824	2.05	11478	1.78	14.47
10031	540023	1	007102	3	1	2	1,4,9	4	3291	2.12	11781	1.80	8.78
10032	470115	13	028601	3	9	1	2	3	1500	0.71	13052	.54	4.99
10033	211115	8	008402	3	9	1	2	3	1500	0.71	12463	.56	12.75
10034	211063	8	016200	3	9	1	2	3	1500	1.41	12270	1.15	21.11
10035	170021	10	048408	3	1	2	2	3	10000	3.69	10495	3.52	4.00
10036	170145	10	090002	3	2	2	1	3	10000	3.67	10535	3.48	10.06
10036	170145	10	090002	3	2	2	1,4,9	4	8332	3.17	10718	2.96	11.86
10037	300003	22	069802	2	2	1	1	3	10000	0.41	9653	.42	17.77
10037	300003	22	069802	2	2	1	9	2	-----	0.36	9444	.38	18.58
10038	560007	19	036555	2	2	1	1	3	10000	0.56	9758	.57	15.98
10038	560007	19	036555	2	2	1	1,4	4	8746	0.82	10308	.79	46.12
10039	560007	19	081700	2	2	1	1	3	10000	0.69	10665	.65	31.78
10039	560007	19	081700	2	2	1	1,4	4	9056	1.04	10585	.98	42.57
10040	560003	19	078300	2	2	1	1	3	10000	0.64	9314	.68	15.32
10041	190037	19	100007	2	2	1	1	3	10000	0.73	9582	.76	7.40
10042	170117	10	048408	3	1	2	1	3	6000	3.69	10565	3.49	2.13
10042	170117	10	048408	3	1	2	1,4	4	8139	3.53	10581	3.34	11.93
10043	170057	10	048905	3	2	2	1	3	2000	2.56	10373	2.47	3.55
10064	380057	21	056100	1	2	1	1	4	100000	0.55	6665	.82	33.28
10065	170163	10	048408	3	1	2	1	4	3752	3.25	10606	3.13	6.16
10065	170163	10	048408	3	1	2	1	3	3856	3.27	10608	3.15	5.90

									AVERAGE CHARACTERISTICS				
Mine Code (1)	FIPS Code (2)	USBM Dist. (3)	Bed Code (4)	Coal Rank (5)	Mining Method (6)	Preparation Code (7)	Method of Sampling (8)	Type of Sample (9)	Volume (Tons) (10)	Sulfur Content (% A.R.) (11)	Heat Content (Btu/lb A.R.) (12)	Lbs Sulfur/ 10^6Btu (13)	RSD of Lbs Sulfur/ 10^6Btu (14)
10046	170157	10	048408	3	1	2	1	4	1378	3.48	10567	3.29	7.20
10047	540023	1	100008	3	2	1	1,4	4	4958	1.99	11060	1.80	16.00
10048	211013	8	100009	3	2	1	9	4	6251	1.35	12456	1.08	26.01
10049	212183	9	090000	3	4	1	9	3	1390	2.96	11623	2.56	28.09
10050	211235	8	015703	3	4	1	9	3	2133	0.97	13466	.72	42.92
10051	211121	8	015703	3	4	1	9	3	5964	1.09	12763	.85	19.50
10052	212177	9	100010	3	4	2	9	3	16916	3.11	11480	2.70	7.33
10053	422005	1	095203	3	1	1	9	4	11370	2.27	11916	1.90	10.94
10054	422063	1	007102	3	1	1	9	4	5306	2.39	11574	2.07	11.06
10055	422063	1	007102	3	1	1	9	4	2685	2.52	11594	2.18	28.01
10056	212---	9	048913	3	2	9	9	9	1363	3.37	10364	3.27	15.03
10057	080107	17	009900	3	2	9	9	4	6838	0.59	11035	.53	6.49
10058	170041	10	048408	3	1	2	9	4	1967	3.03	10793	2.81	13.06
10059	380057	21	056901	1	2	1	1	4	2254	0.79	7065	1.12	25.20
10060	380011	21	056400	1	2	1	1	4	10227	0.80	6115	1.30	25.93
10061	560005	19	092600	2	2	1	1	3	10467	0.33	8764	.37	14.12
10061	560005	19	092600	2	2	1	1	3	10715	0.36	8797	.38	12.06
10062	190135	12	051700	3	1	1	4	4	1377	3.44	9362	3.70	21.51
10062	100135	12	051700	3	1	1	9	9	8627	3.04	9539	3.18	18.08
10063	290175	15	049202	3	2	1	9	9	51280	5.13	8846	5.80	3.10
10064	290121	15	049202	3	2	1	9	9	57120	3.87	10269	3.76	4.81
10065	190117	12	051700	3	1	1	4	4	788	2.25	8899	2.53	20.56
10065	190117	12	051700	3	1	1	9	9	3646	3.07	8969	3.42	19.13
10066	290171	15	048414	3	2	2	9	9	6619	2.72	10252	2.66	20.92
10067	190179	12	053004	3	2	1	9	9	4083	5.76	9666	5.99	16.86
10068	190123	12	------	3	2	1	9	9	1313	4.24	9501	4.46	13.25
10069	190123	12	------	3	2	1	9	9	3596	5.55	9181	6.12	18.19
10070	190125	12	------	3	2	1	9	9	450	4.18	8933	4.68	16.88
10071	040017	18	100011	2	2	1	1	5	12267	0.41	10574	.38	15.55
10072	211193	8	100012	3	4	1	4	3	1500	0.74	12337	.60	36.36
10073	211071	8	100013	3	4	1	4	3	1500	0.78	12126	.65	48.30
10074	211---	8	100014	3	4	1	4	3	100	0.74	12262	.61	13.76
10075	212101	9	048913	3	2	1	1	3	1422	4.25	9890	4.30	10.11
10076	560005	19	095100	2	1	1	1	3	10000	0.40	8244	.48	9.64
10076	560005	19	095100	2	1	1	4	4	8632	0.53	8282	.64	15.58
10077	560007	19	100017	2	1	1	1	3	2959	0.92	10356	.88	17.00
10077	560007	19	100017	2	2	1	4	4	3442	1.08	10017	1.08	17.21
10077	560007	19	100017	2	2	1	1,6	4	9342	1.06	10362	1.02	24.64
10078[c]	080123	16	076800	2	1	1	--	--	-----	----	-----	----	-----
10079[c]	080057	16	004900	2	2	1	--	--	-----	----	-----	----	-----
10079	080057	16	004900	2	2	1	4	3	741	0.21	10803	.19	20.81
10080	422033	1	100015	3	2	2	1	4	1279	2.06	12405	1.64	21.11
10081	422033	1	100015	3	2	2	1	4	741	1.70	12354	1.38	26.24
10082	422005	1	100016	3	2	2	1	4	907	1.19	12776	.93	30.00
10083	540049	3	003606	3	1	2	1	4	854	2.43	13157	1.85	6.29
10084	540049	3	003606	3	1	2	1	4	1032	2.49	13111	1.90	11.65
10085	422033	1	100016	3	2	1	1	4	571	1.56	12623	1.23	25.94
10086	422051	2	100031	3	2	1	1	4	1083	2.06	12666	1.61	14.72

									AVERAGE CHARACTERISTICS				
Mine Code (1)	FIPS Code (2)	USBM Dist. (3)	Bed Code (4)	Coal Rank (5)	Mining Method (6)	Preparation Code (7)	Method of Sampling (8)	Type of Sample (9)	Volume (Tons) (10)	Sulfur Content (% A.R.) (11)	Heat Content (Btu/lb A.R.) (12)	lbs Sulfur/ MMBtu (13)	RSD of lbs Sulfur/ MMBtu (%) (14)
10087	211159	8	016811	3	1	2	1	3	9199	0.67	12985	.51	9.49
10088	080107	17	075000	3	2	1	4	4	1983	0.61	10721	.57	10.36
10089	080081	17	076900	3	2	1	4	4	4133	0.37	10479	.35	20.69
10090	080071	17	075701	3	2	1	4	4	2232	0.60	11132	.54	5.05
10091	080081	17	074502	3	1	1	4	4	4959	0.48	10101	.47	11.51
10092	170145	10	048408	3	2	2	1,4,9	4	8568	2.87	11056	2.60	10.07
10093	170145	10	090002	3	2	2	1,4	4	9128	2.92	10939	2.67	9.16
10094	170077	10	048408	3	2	2	1,4,9	4	8128	1.33	10796	1.24	51.03
10095	170117	10	048408	3	1	2	1,4,9	4	8069	3.19	10689	2.99	14.07
10096	212183	9	050609	3	2	2	4,9	4	1386	1.87	11840	1.60	47.24
10097	170145	10	090002	3	2	2	1,4,9	4	8945	3.03	10900	2.78	10.97
10097	170145	10	090002	3	2	2	1	3	9288	3.30	10974	3.00	6.74
10098	350031	18	047800	2	2	1	1	3	6162	0.44	10156	.43	11.70
10098	350031	18	047800	2	2	1	1,4	4	8716	0.75	10402	.73	45.80
10099	560007	19	039800	2	2	1	1,4	4	9162	0.74	9997	.75	18.63
10100	180125	11	100018	3	2	2	4	4	1379	1.22	11095	1.10	17.09
10101	170055	10	048408	3	1	2	4	4	1317	1.21	11998	1.02	15.68
10102	170055	10	048408	3	1	1	1,4,9	4	8103	2.73	11076	2.48	22.35
10103	170081	10	048408	3	1	2	4	4	1316	1.90	11514	1.66	28.52
10104	170081	10	048408	3	1	2	1,4,9	4	3384	1.51	11806	1.28	12.92
10105	170163	10	048408	3	1	2	4	4	1416	3.29	10821	3.05	8.94
10106	080081	17	100021	3	2	1	1,4	4	6364	0.56	9628	.58	39.82
10107	490007	20	100022	3	1	2	1,4	4	8360	1.01	11204	.90	27.63
10108	510027	8	100023	3	1	2	4	4	1446	1.12	13184	.85	11.57
10109	170057	10	048408	3	1	2	1,4	4	8730	2.90	11039	2.63	9.79
10110	211000	8	100024	3	2	1	1	4	5959	1.43	11880	1.20	30.14
10111	211193	8	100025	3	2	1	1	4	6411	1.37	11003	1.25	44.01
10112	211095	8	100026	3	2	1	1	4	6274	1.15	11997	.96	13.38
10113	390119	4	------	3	2	1	1,4	4	1183	4.04	11020	3.67	31.33
10114	390119	4	003607	3	2	1	1,4	4	1192	3.88	11038	3.52	17.05
10115	390031	4	008013	3	2	1	1,4	4	3361	4.67	10767	4.35	21.71
10116	390031	4	008013	3	1	1	1,4	4	2373	4.62	10096	4.59	19.15
10117	390031	4	008013	3	1	1	1,4	4	947	5.00	10073	4.99	18.86
10118	300059	4	------	3	2	1	1,4	4	1382	3.52	11151	3.16	16.84
10119	390127	4	008002	3	2	1	1,4	4	1192	3.76	10989	3.42	22.34
10120	390031	4	002302	3	2	1	1,4	4	1294	3.95	11135	3.55	15.82
10121	390157	4	100027	3	2	1	1,4	4	1150	4.44	11273	3.95	19.47
10122	390157	4	008404	3	2	1	1,4	4	1426	4.29	11243	3.82	21.19
10123	390031	4	008424	3	2	1	1,4	4	1388	4.27	11340	3.76	16.99
10124	390031	4	008424	3	2	1	1,4	4	1044	4.10	11133	3.69	23.39
10125	390157	4	008424	3	2	1	1,4	4	1213	3.74	11151	3.35	16.65
10126	390031	4	------	3	2	1	1,4	4	959	4.18	11177	3.75	24.51
10127	390079	4	100028	3	2	1	1,4	4	1345	4.10	11414	3.61	18.86
10128	390157	4	100029	3	2	1	1,4	4	1248	3.91	11165	3.50	20.42
10129	390067	4	------	3	2	1	1,4	4	1311	3.58	11130	3.22	19.64
10130	390081	4	003606	3	2	1	1,4	4	1495	3.52	11081	3.16	22.13
10131	390031	4	------	3	2	1	1,4	4	1181	4.11	10988	3.75	14.77
10132	390000	4	------	3	2	1	1,4	4	1143	3.99	11366	3.49	12.24
10133	390119	4	008002	3	2	1	1,4	4	1568	3.59	11049	3.24	21.07
10134	390157	4	008002	3	2	1	1,4	4	1190	3.92	11323	3.47	22.26
10135	390000	4	------	3	2	1	1,4	4	1056	3.73	11387	3.27	[illegible]
10136	390000	4	------	3	2	1	1,4	4	1170	4.11	11219	3.66	14.22
10137	390031	4	008013	3	2	1	1,4	4	1250	4.52	10721	4.26	22.56
10138	390177	4	------	3	2	1	1,4	4	1360	4.40	11192	3.94	16.51
10139	390000	4	------	3	2	1	1,4	4	1024	2.82	11372	2.47	8.66
10140	390000	4	------	3	2	1	1,4	4	1095	3.98	11563	3.45	5.93

Mine Code (1)	FIPS Code (2)	USBM Dist. (3)	Bed Code (4)	Coal Rank (5)	Mining Method (6)	Preparation Code (7)	Method of Sampling (8)	Type of Sample (9)	AVERAGE CHARACTERISTICS Volume (Tons) (10)	Sulfur Content (% A.R.) (11)	Heat Content (Btu/lb A.R.) (12)	Lbs Sulfur/ MMBtu (13)	ESD of Lbs Sulfur/ MMBtu (%) (14)
10141[b]	390000	4	------	3	2	1	1,4	4	--	--	--	--	--
10142	390163	4	008701	3	2	1	1,4	4	989	4.19	11061	3.80	15.15
10143	390163	4	------	3	2	1	1,4	4	984	3.94	11089	3.56	18.85
10144	390073	4	007406	3	2	1	1,4	4	919	3.64	11242	3.25	19.57
10145	390163	4	------	3	2	1	1,4	4	960	3.79	11109	3.43	24.66
10146	390073	4	008013	3	2	1	1,4	4	779	3.47	11055	3.17	22.71
10147	390127	4	008013	3	1	2	1,4	4	992	2.68	11215	2.39	12.07
10148	390163	4	088501	3	2	1	1,4	4	849	3.87	11057	3.53	26.60
10149	390073	4	------	3	2	1	1,4	4	876	3.68	11449	3.20	7.06
10150	390073	4	088701	3	2	1	1,4	4	920	2.99	11008	2.72	14.74
10151[b]	390000	4	------	3	2	1	1,4	4	--	--	--	--	--
10152[b]	390000	4	------	3	2	1	1,4	4	--	--	--	--	--
10153	390000	4	------	3	2	1	1,4	4	1371	1.96	10774	1.86	9.28
10154	390000	4	------	3	2	1	1,4	4	1273	2.58	10424	2.49	20.41
10155	390000	4	------	3	2	1	1,4	4	1276	4.53	9525	4.75	0.00
10156	390000	4	------	3	2	1	1,4	4	128	4.47	9770	4.57	0.00
10157	390127	4	------	3	2	1	1,4	4	990	3.17	10937	2.90	10.46
10158	390059	4	085801	3	2	1	1,4	4	1148	3.49	11061	3.16	26.95
10159	390119	4	------	3	2	1	1,4	4	1245	3.71	10703	3.48	15.09
10160	390127	4	------	3	2	1	1,4	4	1046	3.36	11017	3.07	21.06
10161	390031	4	------	3	2	1	1,4	4	1449	3.81	10593	3.60	7.36
10162	390127	4	------	3	2	1	1,4	4	2025	0.85	10650	.79	0.00
10163[b]	390157	4	------	3	2	1	1,4	4	--	--	--	--	--
10164	390115	4	------	3	2	1	1,4	4	1446	4.00	10962	3.71	20.62
10165	390079	4	------	3	2	1	1,4	4	801	3.64	10664	3.43	17.81
10166[b]	390000	4	------	3	2	1	1,4	4	--	--	--	--	--
10167	390163	4	------	3	2	1	1,4	4	1040	3.15	10580	2.97	0.00
10168	390000	4	------	3	2	1	1,4	4	883	4.49	11572	3.90	15.02
10169[b]	390000	4	------	3	2	1	1,4	4	--	--	--	--	--
10170	390000	4	------	3	2	1	1,4	4	1526	4.60	11296	4.08	16.56
10171	390119	4	008002	3	2	1	1,4	4	924	3.57	11186	3.18	8.82
10172	390000	4	------	3	2	1	1,4	4	398	4.35	10472	4.24	36.11
10173	390127	4	------	3	2	1	1,4	4	512	3.67	11176	3.29	14.72
10174	390031	4	092801	3	3	1	1,4	4	1016	3.54	11772	3.00	3.15
10175	390000	4	100030	3	4	1	1	5	3757	4.05	10645	3.80	16.38
10175	390000	4	100030	3	4	1	1	5					
10175	390000	4	100030	3	4	1	1	5'					
10175	390000	4	100030	3	4	1	4	5					
10176	390000	4	100030	3	4	1	4	5	2443	3.40	11329	3.00	15.76
10177	390000	4	100030	3	4	1	1	5	1696	3.82	10985	3.48	21.48
10178	300003	22	100033	2	2	1	1	3	10621	0.65	8625	.75	12.42
10179	380065	21	100034	1	2	1	2	1	----	0.76	6134	1.23	46.03
10180	------	8	------	3	9	9	9	5	5595	0.69	11683	.59	18.61
10181	------	8	------	3	9	9	9	5	26411	0.97	12219	.79	11.56
10182	211---	8	------	3	9	9	9	9	6789	1.01	11916	.85	15.39
10183	212177	9	100010	3	2	1	1	4	7326	3.97	10239	3.89	14.53
10184	180173	11	048305	3	2	2	1	3	5525	3.14	11039	2.85	8.29
10185	180021	11	050202	3	2	2	1	2	6625	4.07	10989	3.70	10.70
10186	212177	9	100035	3	2	2	1	3	1466	2.23	11006	2.03	12.41
10187	180153	11	100036	3	2	2	1	2	8050	1.70	10810	1.57	34.36
10188	170185	10	048905	3	1	1	1	2	6056	1.53	10657	1.43	12.72
10189	180173	11	048905	3	2	1	1	3	4563	3.56	10573	3.37	13.96
10190	170057	10	049608	3	2	2	1	3	4701	2.84	11163	2.54	10.91
10191	211119	8	011108	3	2	1	1	4	4350	1.20	11098	1.08	28.25
10192	390013	4	085802	3	2	1	1	4	3104	2.96	10565	2.78	15.41

									AVERAGE CHARACTERISTICS				
Mine Code (1)	FIPS Code (2)	USBM Dist. (3)	Bed Code (4)	Coal Rank (5)	Mining Method (6)	Preparation Code (7)	Method of Sampling (8)	Type of Sample (9)	Volume (Tons) (10)	Sulfur Content (% A.R.) (11)	Heat Content (Btu/lb A.R.) (12)	Lbs Sulfur/ 10^6Btu (13)	RSD of Lbs Sulfur/ 10^6Btu (%) (14)
10193	540039	8	008402	3	2	1	1	4	3111	0.94	11188	.84	18.14
10194 b/	540039	22	008402	3	2	1	1	4	--	--	--	--	--
10195 b/	300083	22	------	1	--	1	9	1	--	--	--	--	--
10196	300083	22	034500	1	2	1	9	2	--	0.55	6478	.84	47.73
10197 b/	300017	22	------	2	--	1	9	1	--	--	--	--	--
10198 b/	300017	22	------	2	--	1	9	1	--	--	--	--	--
10199 b/	300017	22	------	2	--	1	9	1	--	--	--	--	--
10200 b/	300017	22	------	2	--	1	9	1	--	--	--	--	--
10201 b/	300087	22	------	2	--	1	9	1	--	--	--	--	--
10202	300087	22	093100	2	2	1	9	2	--	0.75	9042	.83	19.87
10203 b/	300003	22	093100	2	2	1	9	1	--	--	--	--	--

a/ For interpretation of codes in columns (1)-(9) refer to Appendix C.
b/ Data set not analyzed.
c/ Coal source represented by stack monitoring data, no fuel analyses available.
d/ Dry basis.
e/ Data sets 10206 through 10331 analyzed on USBM District basis only, due to lack of seam information and small number of analyses.

Source: Foster Associates, Inc.

APPENDIX I

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY INDIVIDUAL DATA SETS
(FIRST RUN: DATA AS RECEIVED FROM RESPONDENTS, DISREGARDING LOT-SIZE ANALYSIS)

U.S.B.M. PRODUCING DISTRICT (1)	SEAM CODE (2)	MINE CODE (3)	VOLUME SAMPLED (Tons) Average (4)	VOLUME SAMPLED (Tons) RSD(%) (5)	SULFUR CONTENT (Percent, As Received) Average (6)	SULFUR CONTENT (Percent, As Received) RSD(%) (7)	HEAT CONTENT (Btu/lb., As Required) Average (8)	HEAT CONTENT (Btu/lb., As Required) RSD(%) (9)	LBS SULFUR/MMBtu Average (10)	LBS SULFUR/MMBtu RSD(%) (11)	NUMBER OF ANALYSIS (12)
1	001000	10017	2055	64.4	1.61	34.0	11078	4.4	1.45	32.6	55
1	007102	10031	2824	50.6	2.04	14.2	11478	2.2	1.78	14.5	268
1	007102	10031	3291	61.7	2.12	8.0	11781	2.1	1.80	8.8	220
1	100008	10047	4958	88.2	1.99	15.7	11060	3.9	1.80	16.0	215
1	095203	10053	11370	41.0	2.27	10.0	11916	2.5	1.91	10.9	2039
1	007102	10054	5306	31.7	2.39	10.5	11574	2.0	2.07	11.1	1279
1	007102	10055	2685	36.0	2.52	23.9	11594	4.3	2.18	26.0	1033
1	100015	10080	1279	89.9	2.04	22.0	12405	2.0	1.64	21.1	11
1	100015	10081	741	38.7	1.70	23.8	12354	3.8	1.38	26.2	17
1	100016	10082	907	65.0	1.19	30.0	12776	2.9	0.93	30.0	28
1	100016	10085	571	47.0	1.54	24.9	12623	4.0	1.23	25.9	19
2	100031	10086	1083	78.4	2.04	14.3	12666	3.2	1.61	14.7	21
3	003604	10029a/	11870	31.6	2.80	7.9	13054	1.7	2.14	8.3	704
3		10083b/									
3	003604	10084	1032	66.0	2.49	10.1	13111	2.8	1.90	11.8	42
4	007402	10030a/	10360	10.0	2.60	5.0	12481	5.6	2.09	6.6	275
4	------	10113	1103	32.0	4.04	31.1	11030	6.5	3.67	31.3	127
4	003607	10114	1192	33.5	3.88	17.3	11038	5.3	3.51	17.0	94
4	008013	10115	3361	30.9	4.67	20.9	10763	5.7	4.35	21.7	417
4	008013	10116	2373	59.8	4.62	17.0	10096	4.8	4.59	19.1	427
4	008013	10117	946	47.5	5.00	16.1	10073	6.3	4.99	18.9	130
4	------	10118	1302	21.7	3.52	17.3	11151	3.9	3.15	16.0	495
4	008002	10119	1192	29.2	3.91	70.5	11003	5.9	3.55	67.2	278
4	002302	10120	1294	22.7	3.95	15.5	11135	4.1	3.55	15.8	516
4	100027	10121	1149	38.3	4.44	17.7	11273	4.0	3.95	19.5	63
4	008404	10122	1426	22.1	4.29	20.4	11243	4.3	3.82	21.2	351
4	008424	10123	1388	21.9	4.27	16.2	11340	4.1	3.76	17.8	251
4	008424	10124	1044	36.6	4.10	21.6	11133	4.0	3.69	23.4	102
4	008404	10125	1212	26.3	3.74	16.7	11151	4.1	3.35	16.6	322
4	------	10126	959	29.7	4.18	22.4	11177	4.9	3.70	24.5	29
4	100028	10127	1345	31.3	4.10	15.6	11414	5.3	3.61	18.9	164
4	100029	10128	1248	27.6	3.91	20.2	11165	4.6	3.51	20.6	165
4	------	10129	1311	24.1	3.58	19.5	11131	4.1	3.22	19.6	223
4	003604	10130	1495	30.2	3.51	23.4	11081	5.3	3.16	22.1	110
4	------	10131	1180	31.2	4.10	12.4	10988	4.9	3.75	14.8	37
4	------	10132	1143	46.7	3.99	14.9	11366	3.0	3.49	12.2	5
4	008002	10133	1568	23.8	3.58	21.5	11049	5.0	3.24	21.1	461
4	008002	10134	1190	36.9	3.92	19.6	11323	2.8	3.47	22.3	42
4	------	10135	1056	19.8	3.73	21.0	11387	2.6	3.27	20.3	19
4	------	10136	1170	18.2	4.11	13.9	11218	3.2	3.66	14.2	9
4	008013	10137	1250	60.1	4.52	19.0	10721	9.2	4.26	22.6	37
4	------	10138	1360	30.1	4.40	14.8	11192	5.1	3.94	16.5	10
4	------	10139	1024	45.0	2.82	11.1	11372	4.5	2.47	8.6	5
4	------	10140	1096	20.0	3.98	5.2	11563	2.5	3.45	5.9	4
4	------	10141b/									
4	008701	10142	909	28.0	4.19	13.6	11061	3.9	3.80	15.3	208
4	------	10143	984	20.0	3.94	17.2	11089	3.8	3.56	18.8	251
4	007406	10144	919	28.3	3.64	17.8	11242	3.5	3.25	19.6	152
4	------	10145	959	23.5	3.79	21.3	11109	5.6	3.43	24.7	174
4	008013	10146	779	40.4	3.47	18.1	11055	6.1	3.17	22.7	39
4	008013	10147	992	17.1	2.68	11.8	11215	1.7	2.39	12.1	261
4	088501	10148	849	35.2	3.87	23.0	11057	5.1	3.53	26.6	77
4	------	10149	876	16.4	3.68	9.5	11269	2.5	3.21	7.1	2
4	088701	10150	920	16.5	2.99	13.6	11008	2.8	2.72	14.7	13
4	------	10151b/									
4	------	10152b/									
4	------	10153	1371	22.5	1.96	10.0	10774	3.4	1.82	9.3	7
4	------	10154	1273	29.2	2.57	16.5	10424	5.3	2.49	20.4	13
4	------	10155	1276	0.0	4.51	0.0	9525	0.0	4.75	0.0	1

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY INDIVIDUAL DATA SETS
(FIRST RUN: DATA AS RECEIVED FROM RESPONDENTS, DISREGARDING LOT-SIZE ANALYSIS)

U.S.B.M. PRODUCING DISTRICT	SEAM CODE	MINE CODE	VOLUME SAMPLED (Tons)		SULFUR CONTENT (Percent, As Received)		HEAT CONTENT (Btu/lb., As Required)		LBS SULFUR/MMBtu		NUMBER OF ANALYSIS
			Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4	------	10156	128	0.0	4.47	0.0	9770	0.0	4.57	0.0	1
4	------	10157	990	4.3	3.37	10.1	10937	2.0	2.91	10.5	6
4	085001	10158	1148	37.7	3.49	25.8	11061	5.0	3.16	26.9	27
4	------	10159	1245	33.9	3.71	11.7	10703	7.0	3.48	15.1	45
4	------	10160	1046	37.4	3.36	17.7	11017	5.1	3.07	21.0	50
4	------	10161	1449	4.0	3.81	4.1	10593	4.1	3.60	7.3	6
4	------	10162	2025	0.0	0.85	0.0	10650	0.0	0.79	0.0	1
4	------	10163b/									
4	------	10164	1446	18.2	4.00	10.6	10962	9.2	3.71	20.6	5
4	------	10165	801	38.1	3.64	16.3	10664	3.8	3.43	17.8	28
4	------	10166b/									
4	------	10167	1040	0.0	3.15	0.0	10580	0.0	2.97	0.0	1
4	------	10168	803	40.0	4.49	12.3	11572	3.5	3.90	15.0	10
4	------	10169b/									
4	------	10170	1526	10.8	4.60	12.9	11296	1.7	4.08	14.5	2
4	008002	10171	924	14.7	3.57	10.7	11186	2.6	3.18	8.8	5
4	------	10172	398	48.1	4.35	29.7	10472	5.9	4.24	36.1	5
4	------	10173	512	28.3	3.67	13.7	11176	1.0	3.29	14.7	2
4	092801	10174	1016	4.5	3.54	3.4	11772	0.2	3.01	3.1	2
4	100030	10175	3757	51.3	4.05	15.9	10645	5.1	3.81	16.4	1564
4	100030	10176	2443	49.2	3.40	15.0	11329	3.0	3.01	15.8	398
4	100030	10177	1696	49.6	3.82	21.1	10985	3.3	3.48	21.5	286
4	085002	10192	3104	49.0	2.94	15.1	10565	3.7	2.78	15.4	47
8	010401	10001	5318	24.7	2.69	22.3	11650	1.8	2.31	21.6	45
8	015109	10002	3150	52.8	1.34	22.5	12423	4.4	1.08	23.8	22
8	100001	10003	5680	18.5	2.18	23.2	11440	3.4	1.91	23.9	73
8	100004	10016	10024	11.4	1.48	10.9	11836	3.6	1.25	10.5	30
8	100005	10022	5060	55.8	0.61	5.0	12301	1.3	0.49	4.9	113
8	015104	10023	2594	56.7	0.73	9.9	11329	2.6	0.64	8.9	115
8	100006	10025	1800	c/	0.85	19.3	12404	4.7	0.69	20.8	55
8	015108	10026	1100	c/	1.13	58.1	12195	5.3	0.94	62.3	51
8	021202	10027	1600	c/	0.98	42.9	12936	3.3	0.76	45.2	11
8	011108	10028	300	c/	0.68	12.5	12620	3.8	0.54	14.4	10
8	008402	10033	1500	c/	0.71	11.2	12463	4.4	0.56	12.7	10
8	016200	10034	1500	c/	1.41	20.6	12270	2.2	1.15	21.1	40
8	100009	10048	6251	12.0	1.35	24.2	12456	2.8	1.08	26.0	119
8	015703	10050	2133	27.2	0.97	41.9	13466	3.5	0.72	42.9	173
8	015703	10051	5964	37.4	1.09	18.7	12763	3.9	0.85	19.5	219
8	100012	10072	1500	c/	0.74	35.4	12317	4.9	0.60	34.4	68
8	100013	10073	1500	c/	0.78	47.7	12126	6.2	0.65	48.3	50
8	100014	10074	100	c/	0.74	12.0	12242	3.2	0.61	13.8	6
8	016811	10087	9199	13.1	0.67	9.6	12985	1.0	0.51	9.5	57
8	100023	10108	1446	44.0	1.11	10.6	13184	2.9	0.85	11.6	26
8	100024	10110	5959	26.5	1.43	29.5	11880	2.5	1.20	30.1	29
8	100025	10111	6411	6.9	1.37	41.7	11004	3.6	1.25	44.0	59
8	100026	10112	6274	5.5	1.15	12.5	11997	2.2	0.96	13.4	66
8	------	10180	5495	19.6	0.69	18.8	11683	3.0	0.59	18.6	701
8	------	10181	26411	27.2	0.97	10.8	12219	2.7	0.79	11.6	64
8	------	10182	6789	254.3	1.01	34.2	11916	4.4	0.85	34.4	977
8	011108	10191	4350	35.3	1.20	28.6	11098	4.1	1.08	28.2	89
8	008402	10193	3111	51.4	0.94	16.7	11188	4.5	0.84	18.1	30
9	048913	10009	6254	27.7	3.82	8.0	11008	2.4	3.47	9.1	1026
9	048415	10018	4548	12.8	4.19	4.4	11054	2.3	3.80	5.7	182
9	048415	10019	3739	14.3	3.65	4.2	10760	2.5	3.39	4.9	114
9	048415	10019	5643	37.0	3.03	14.1	11197	3.5	2.71	13.4	31
9	048415	10020	4031	14.6	2.99	4.4	10965	1.4	2.73	5.0	75
9	048415	10021	4354	17.7	3.46	4.7	10577	1.7	3.27	5.3	126
9	090000	10049	1390	7.1	2.96	25.5	11623	3.7	2.56	28.1	417

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY INDIVIDUAL DATA SETS
(FIRST RUN: DATA AS RECEIVED FROM RESPONDENTS, DISREGARDING LOT-SIZE ANALYSIS)

U.S.B.M. PRODUCING DISTRICT (1)	SEAM CODE (2)	MINE CODE (3)	VOLUME SAMPLED (Tons) Average (4)	VOLUME SAMPLED (Tons) RSD(%) (5)	SULFUR CONTENT (Percent, As Received) Average (6)	SULFUR CONTENT (Percent, As Received) RSD(%) (7)	HEAT CONTENT (Btu/lb., As Received) Average (8)	HEAT CONTENT (Btu/lb., As Received) RSD(%) (9)	LBS. SULFUR/MMBtu Average (10)	LBS. SULFUR/MMBtu RSD(%) (11)	NUMBER OF ANALYSIS (12)
9	100010	10052	16916	41.7	3.11	6.1	11480	2.9	2.71	7.3	131
9	048913	10056	1343	5.6	3.37	13.2	10344	4.2	3.27	15.0	288
9	048913	10075	1422	28.1	4.25	8.6	9890	2.8	4.31	10.1	43
9	050609	10096	1386	13.5	1.87	45.5	11840	4.4	1.60	47.2	41
9	100010	10183	7326	30.0	3.97	13.8	10239	4.5	3.89	14.5	254
9	100035	10186	1466	5.5	2.23	13.6	11006	2.3	2.03	12.4	1231
10	090002	10004	9110	8.4	2.24	10.7	10588	2.5	2.11	10.5	326
10	090002	10024	1470	7.4	3.49	11.8	10990	3.3	3.17	11.2	166
10	048408	10035	10000	c/	3.69	3.8	10495	1.7	3.52	4.0	63
10	090002	10036	10000	c/	3.67	8.5	10535	3.3	3.48	10.1	107
10	090002	10036	8332	10.3	3.17	11.6	10718	3.1	2.96	11.8	61
10	048408	10042	6000	c/	3.69	2.0	10565	0.7	3.49	2.1	147
10	048408	10042	8139	8.4	3.53	11.0	10581	3.1	3.34	11.9	77
10	048905	10043	2000	c/	2.56	3.5	10373	1.4	2.47	3.6	48
10	048408	10045	3752	60.1	3.25	5.0	10404	2.4	3.13	6.2	214
10	048408	10045	3856	60.0	3.27	5.5	10408	2.3	3.15	5.9	213
10	048108	10046	1378	5.4	3.48	6.5	10567	2.1	3.29	7.2	599
10	048408	10058	1967	26.0	3.03	11.6	10793	3.0	2.81	13.3	211
10	048408	10092	8568	11.5	2.83	10.1	11056	2.6	2.59	10.1	34
10	090002	10093	9128	16.1	2.92	9.5	10939	2.5	2.67	9.4	320
10	048408	10094	8128	21.0	1.33	50.8	10796	4.3	1.24	51.0	317
10	048408	10095	8049	22.3	3.19	13.2	10689	3.4	2.99	14.1	142
10	090002	10097	8945	17.3	3.03	11.3	10900	3.2	2.78	11.0	139
10	090002	10097	9288	15.8	3.30	6.6	10974	1.5	3.00	6.7	269
10	048408	10101	1317	12.8	1.21	15.1	11998	2.1	1.02	15.7	38
10	048408	10102	8103	12.7	2.73	20.4	11076	4.7	2.48	22.3	163
10	048408	10103	1316	21.8	1.90	25.5	11514	2.8	1.66	28.5	19
10	048408	10104	3384	92.7	1.51	12.7	11804	2.9	1.28	12.9	170
10	048408	10105	1416	43.1	3.29	8.2	10821	2.7	3.04	8.9	54
10	048408	10109	8729	14.5	2.90	10.9	11039	2.6	2.63	9.8	19
10	048905	10188	6056	34.5	1.53	12.5	10657	4.2	1.43	12.7	238
10	049608	10190	4701	35.5	2.84	10.7	11163	1.1	2.54	10.9	152
11	100018	10100	1379	6.7	1.22	16.4	11095	2.5	1.11	17.1	74
11	048305	10184	5525	62.0	3.14	7.7	11039	1.5	2.85	8.3	362
11	050202	10185	6425	40.9	4.07	10.0	10989	2.0	3.71	10.7	234
11	100036	10187	8050	23.1	1.70	34.0	10810	1.7	1.57	34.3	233
11	048905	10189	4542	62.5	3.56	12.8	10571	2.6	3.37	14.0	335
12	051700	10062	10345	133.7	3.04	17.8	9539	3.2	3.18	18.1	55
12	051700	10062	1377	26.4	3.44	17.9	9342	4.6	3.71	21.5	53
12	051700	10065	788	68.1	2.25	18.8	8899	4.8	2.53	20.5	15
12	051700	10065	3646	54.3	3.07	20.0	8969	3.3	3.42	19.1	27
12	053004	10067	4083	55.4	5.76	14.5	9666	5.1	5.99	16.9	23
12	------	10068	1313	58.1	4.21	11.7	9501	2.2	4.44	13.2	17
12	------	10069	3596	64.1	5.55	12.2	9101	9.9	6.12	18.2	8
12	------	10070	450	48.9	4.18	16.3	8933	4.6	4.68	16.9	5
13	028601	10032	1500	c/	0.71	4.6	13052	1.0	0.54	4.8	17
15	049202	10010	2396	58.6	2.99	11.5	10905	3.7	2.75	12.2	17
15	049202	10011	706	59.6	3.86	9.3	10056	8.7	3.09	16.7	14
15	049008	10012	505	42.6	3.77	11.7	10834	2.1	3.46	12.2	19
15	049202	10013	712	64.2	3.68	5.1	10752	2.5	3.42	5.6	24
15	049202	10014	904	76.1	4.03	12.9	9615	4.9	4.20	14.4	20
15	098700	10015	580	81.6	4.74	11.9	9829	5.6	4.85	14.5	29
15	049202	10063	49760	12.9	5.06	6.0	8896	3.7	5.70	7.7	25
15	049202	10064	58640	29.6	3.94	9.0	10218	2.8	3.06	12.8	25
15	048414	10066	6619	87.0	2.69	21.3	10252	4.0	2.63	22.7	101

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY INDIVIDUAL DATA SETS
(FIRST RUN: DATA AS RECEIVED FROM RESPONDENTS, DISREGARDING LOT-SIZE ANALYSIS)

U.S.B.M. PRODUCING DISTRICT	SEAM CODE	MINE CODE	VOLUME SAMPLED (Tons)		SULFUR CONTENT (Percent, As Received)		HEAT CONTENT (Btu/lb., As Required)		LBS SULFUR/MMBtu		NUMBER OF ANALYSIS
			Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
16		10078[b]									
16		10079[b]									
16	004900	10079	741	95.5	0.21	19.7	10803	2.3	0.19	20.8	73
17	009900	10057	6838	18.8	0.59	6.4	11035	1.2	0.53	6.5	132
17	075000	10088	1984	31.0	0.61	10.7	10721	3.0	0.57	10.3	272
17	076900	10089	4133	38.6	0.37	21.4	10479	3.1	0.35	20.7	92
17	075701	10090	2232	45.8	0.60	5.3	11132	5.0	0.54	5.0	26
17	074502	10091	4959	29.1	0.48	11.3	10100	2.3	0.47	11.5	87
17	100021	10106	6364	16.9	0.56	41.9	9627	4.0	0.58	39.8	19
18	100011	10071	12267	33.0	0.41	12.0	10574	3.5	0.38	15.6	203
18	047800	10098	6162	5.7	0.44	11.3	10156	2.7	0.43	11.7	41
18	047800	10098	8716	13.3	0.75	46.8	10402	6.4	0.73	45.8	55
19	095100	10006	CORE	----	0.41	17.2	8376	1.7	0.48	16.9	1537
19	095100	10006	11035	2.0	0.39	14.5	8308	1.2	0.46	14.2	33
19	095100	10006	10402	4.8	0.36	12.0	8388	1.1	0.43	11.6	49
19	095100	10006	10798	3.5	0.37	14.9	8360	1.9	0.44	14.4	213
19	095100	10006	10513	5.8	0.36	12.2	8404	1.4	0.43	11.9	64
19	095100	10006	10147	22.1	0.36	12.9	8400	1.4	0.42	12.4	1780
19	100003	10007	10800	12.0	0.40	10.3	8298	1.8	0.47	9.9	302
19	100003	10007	10625	5.5	0.38	6.8	8406	1.4	0.45	6.5	10
19	100002	10008	11692	0.7	0.41	7.6	8440	3.0	0.48	6.9	99
19	036555	10038	10000	[c]	0.56	14.6	9758	2.7	0.57	16.0	140
19	036555	10038	8746	7.3	0.82	47.9	10308	4.4	0.79	46.1	29
19	081700	10039	10000	[c]	0.69	31.0	10665	2.1	0.65	31.8	30
19	081700	10039	9056	7.5	1.04	43.0	10585	3.8	0.98	42.6	28
19	078300	10040	10000	[c]	0.64	15.0	9314	1.0	0.68	15.3	206
19	100007	10041	10000	[c]	0.73	6.5	9582	1.6	0.76	7.4	42
19	092600	10061	10467	4.1	0.33	14.1	8764	1.8	0.37	14.2	47
19	092600	10061	10710	3.9	.33	12.0	8797	1.2	0.38	12.0	52
19	095100	10076	10000	[c]	0.40	11.2	8244	2.9	0.48	9.6	10
19	095100	10076	8632	9.9	0.53	36.1	8282	3.0	0.64	35.6	15
19	100017	10077	2959	42.9	0.92	16.8	10356	2.8	0.88	17.0	169
19	100017	10077	3442	48.0	1.08	16.2	10017	2.1	1.08	17.2	74
19	100017	10077	9342	4.8	1.05	26.8	10362	3.4	1.02	24.6	51
19	039800	10099	9162	8.1	0.74	17.7	9997	4.2	0.75	18.6	23
20	100022	10107	8360	22.6	1.01	26.9	11204	2.8	0.90	27.6	44
21	056100	10044	77192	25.8	0.55	33.3	6665	1.1	0.82	33.3	54
21	056901	10059	2254	23.2	0.79	24.9	7045	1.9	1.12	25.2	1216
21	056400	10060	10227	10.1	0.80	25.6	6115	3.0	1.31	25.9	770
21		10179[b]	CORE								
22	080800	10005	8423	4.2	0.74	29.8	8639	2.4	0.85	30.1	586
22	080800	10005[b]									
22	069802	10037	10000	[c]	0.41	17.4	9653	1.0	0.42	17.8	555
22	069802	10037[b]									
22	100033	10178	10421	0.9	0.65	11.8	8625	1.5	0.75	12.4	19

[a] Analyses on "dry" basis
[b] Analysis not performed due to limited number of observations.
[c] Exact volumes were not provided, volume indicates approximate size of each shipment.

Source: Foster Associates, Inc.

APPENDIX J

ANALYSIS OF DATA ON AN AGGREGATE BASIS BY PRODUCING DISTRICT

This Appendix sets out, in detail, the analyses performed on an aggregate basis in order to examine coal sulfur variabilities within individual Producing Districts. In general, for each Producing District the following analyses, with and without lot-size intervals, were performed:

- All coals
- Raw coals
- Washed coals
- Selected coal seams

In addition, the quantity of data for some Producing Districts permitted more detailed analysis of other factors which may be possible sources of coal sulfur variability. These factors are noted in the following discussion.

1.0 Producing District 1 (Pennsylvania)

1.1 Raw and Washed Coals

An aggregate comparison for all coals in Producing District 1 indicated an average of 1.83 lbs S/MMBtu with an RSD of 26.18 percent. As expected, when the raw and washed coals were analyzed separately, both the average lbs S/MMBtu and the RSD were lower for the washed coals. These results are summarized in Table J-1.

TABLE J-1

PRODUCING DISTRICT 1: ALL COALS, RAW, AND WASHED

Type of Coal	Lbs S/MMBtu Average	Lbs S/MMBtu RSD (%)	Number of Observations
All Coals	1.83	26.18	6,619
Raw	1.86	26.22	5,738
Washed	1.65	22.78	881

Source: Schedule J-1, found at end of Appendix J.

As shown in Schedule J-1 at the end of this Appendix, analyses of these data by lot-size intervals exhibit a strong relationship between RSD and lot-size, especially in the case of all coals. An examination of the lot-size analyses for all coals in Producing District 1 shows a steady decline in RSD from 42.5 percent at the 200-500 ton interval to 9.3 percent at the 9,000-10,000 ton interval, while slight increases (to 10.4 percent) are observed in the intervals 11,000-13,000, 13,000-15,000, and 19,000-23,000 tons.

1.2 Upper Freeport Seam, Raw and Washed

Within Producing District 1, a comparison was also made of the raw and washed analyses for the Upper Freeport seam (USBM Code 071). Again, and as expected, both the average lbs S/MMBtu and RSD were lower for the washed coals. For the washed Upper Freeport seam there appears to be no relationship between lot-size and RSD over the tonnage ranges analyzed (0 to 5500 tons).

2.0 Producing District 4 (Ohio)

2.1 Raw and "As-Burned" Coals

In the case of Producing District 4, insufficient data were available for an analysis of washed coals. The raw coals (as delivered) on an aggregate basis exhibit an average of 3.58 lbs S/MMBtu with an RSD of 24.8 percent. As a comparison, analyses for these coals on an "as-burned" or "as-fired" basis indicate an average of 3.62 lbs S/MMBtu with an RSD of 19.0 percent. With the exception of the 200 to 500 ton interval for the "as-burned" coals, both the "as-delivered" and "as-burned" aggregate for raw coals exhibit a general decline in RSD with increasing lot-sizes. As expected, due to the mixing of coals in stockpiles, handling, feeding, etc., the raw "as-burned" coals exhibit lower RSD's or less variation than the raw "as-delivered" coals.

2.2 "As-Burned" Coals by Individual generating Units

The available data for the "as-burned" coals in Producing District 4 permits a more detailed examination of variability. These coals, from the same general sources, were burned in one utility's six generating units. The analyses for units 1, 2, and 3 were reported on a composite basis, while separate analyses were available for units 4, 5, and 6. In general, each observation represents the volume of coal burned during a 24-hour period. The statistics for these units are provided in Table J-2.

TABLE J-2

PRODUCING DISTRICT 4: "AS-BURNED"

Unit Number	Lbs S/MMBtu Average	 RSD (%)	Number of Observations
1, 2, and 3	4.04	15.34	614
4	3.79	16.10	402
5	3.53	14.34	410
6	3.65	16.12	139

Source: Schedule J-1.

The results for the various generating units indicate relatively consistent RSD's ranging from 14 to 16 percent. However, within the units there does not appear to be a relationship between RSD and lot-size, possibly due to the rather limited ranges in lot-size (tons burned/day).

2.3 Pittsburgh, Middle Kittanning, and Lower Kittanning Seams

Within Producing District 4, three seams -- Pittsburgh (036), Middle Kittanning (080), and Lower Kittanning (084) -- were examined on an aggregate basis. These seam data reflect raw coals from various mines in Ohio.

The data for the Pittsburgh seam do not exhibit an inverse relationship between RSD and lot-size. This may be due to the limited number of observations (182) and the relatively narrow range of lot-sizes (600-2100 tons).

The Middle Kittanning and Lower Kittanning seams both exhibit a general decline in RSD with increasing lot-size. A comparison of the aggregate data for these seams is set out in Table J-3. These data indicate that the relationship between RSD and lot-size differs between the seams. The slope of RSD as a function of lot-size is greater for the Middle Kittanning seam, resulting in a relatively greater decrease in RSD for an equal increase in lot-size.

3.0 Producing District 8 (Eastern Kentucky)

In Producing District 8, aggregate comparisons were performed for raw and washed coals and for the Upper Elkhorn No. 3 (151) and Blue Gem (157) seams.

TABLE J-3

COMPARISON OF AGGREGATE DATA FOR THE MIDDLE AND LOWER KITTANNING SEAMS IN OHIO

Lot-Size Interval (Tons)		RSD of Lbs S/MMBtu (%)		Number of Observations
Mid-Point	Range	Middle Kittanning[1]	Lower Kittanning[2]	
440	250-600	--	20.74	29
456	300-600	33.68	--	90
763	600-900	31.57	--	120
1,015	900-1,100	--	20.23	105
1,115	1,000-1,300	31.96	--	426
1,208	1,100-1,300	--	20.44	188
1,407	1,300-1,500	--	19.87	517
1,425	1,300-1,600	26.07	--	394
1,582	1,500-1,800	--	--	91
1,745	1,600-1,900	28.40	--	198
2,137	2,000-2,300	22.61	--	101
2,470	2,000-3,000	--	18.28	22
2,796	2,600-3,000	18.21	--	89
3,441	3,000-4,000	20.56	--	229
4,357	4,000-5,000	21.09	--	142

[1] USBM Code 080
[2] USBM Code 084

Source: Schedule J-1.

3.1 Raw and Washed

The aggregate analyses for all coals in Producing District 8 indicate an average of 0.83 lbs S/MMBtu with an RSD of 47.7 percent. A comparison of the raw and washed coals indicates that the washed coals have a lower lbs S/MMBtu as well as a lower RSD. The raw coals averaged 1.00 lbs S/MMBtu with an RSD of 51.0 percent, while the washed coals averaged 0.65 lbs S/MMBtu with an RSD of 38.5 percent.

An analysis of lot-size for the composite of all coals and raw and washed coals individually failed to demonstrate any significant inverse relationship between RSD and lot-size.

3.2 Upper Elkhorn No. 3 (151) and Blue Gem (157) Seams

Data for these seams show an average of 0.77 and 0.79 lbs S/MMBtu for the Upper Elkhorn No. 3 and Blue Gem seams, respectively. An analysis by lot-size indicates an inverse relationship between RSD and lot-size for both seams. The RSD's for the smallest lot-sizes analyzed for these seams are relatively high. The 900-1400 ton interval for the Upper Elkhorn No. 3 seam indicated an RSD of 60.7 percent, while the 1000-2000 ton interval of the Blue Gem seam indicated an RSD of 42.5.

4.0 Producing District 9 (Western Kentucky)

4.1 Raw and Washed Coals

Analysis of all coals in Producing District 9 yielded an average of 2.84 lbs S/MMBtu with an RSD of 26.8 percent. As expected, the washed coals exhibited both a lower average lbs S/MMBtu and a lower RSD compared to raw coals.

Set out in Table J-4 is a summary of the results obtained from the lot-size analysis of the raw and washed coals in Producing District 9. For both the raw and washed coals, there appears to be a definite inverse relationship between RSD and lot-size. At the smallest lot-size examined (1000-1500 tons), there appears to be a significant difference in the RSD's of the raw and washed coals. The raw coals in this interval exhibit an RSD of 30 percent compared to 15 percent for washed coals. However, at the larger lot-sizes, it appears that the difference between the RSD's of raw and washed coals become less significant.

TABLE J-4

COMPARISON OF RSD VS. LOT-SIZE FOR ALL COALS, RAW COALS, AND WASHED COALS FOR PRODUCING DISTRICT 9

Lot-Size Interval (Tons)		RSD of Lbs S/MMBtu (%)			Number of Observations1/
Mid-Point	Range	All Coals	Raw	Washed	
1,359	1,000-1,500	--	30.39	--	412
1,390	1,000-1.500	29.35	--	--	1,553
1,422	1,000-1,500	--	--	15.05	861
1,549	1,500-2,000	--	--	14.61	427
1,562	1,500-2,000	29.00	--	--	516
1,634	1,500-2,000	--	21.53	--	81
2,479	2,000-3,000	--	11.78	--	46
2,521	2,000-3,000	15.55	--	--	58
3,451	3,000-4,000	8.42	--	--	143
3,454	3,000-4.000	--	7.88	--	140
4,281	4,000-6,000	--	--	9.09	398
4,436	4,000-6,000	11.61	--	--	660
4,516	4,000-5,000	--	8.43	--	225
5,508	5,000-6,000	--	9.16	--	45
6,921	6,000-7,000	--	8.73	--	840
6,924	6,000-8,000	9.01	--	--	865
7,615	6,000-10,000	--	--	9.83	37
14,894	10,000-20,000	--	--	6.64	59
24,047	20,000-30,000	--	--	5.57	49

1/ Excludes all intervals with less than 30 observations.

Source: Schedule J-1.

4.2 No. 11 Seam (484)

The No. 11 seam in western Kentucky, which is equivalent to the No. 6 seam in Illinois, was also examined. A comparison of the raw and washed coals from this seam indicates a lower lbs S/MMBtu and a lower RSD for the washed coals. Sufficient data were not available for an examination of the relationship between RSD and lot-size.

5.0 Producing District 10 (Illinois)

5.1 All Coals: Washed

Data for Producing District 10 are based almost entirely on washed coals. An analysis of all coals yielded an average of 2.65 lbs S/MMBtu with an RSD of 29.5 percent.

Although substantial data were available for a lot-size analysis in the tonnage ranges from 1,000 to 12,000 tons, the results of this analysis do not support the inverse relationship between RSD and lot-size. The lot-size intervals and the calculated RSD's for these coals are set out in Table J-5.

TABLE J-5

COMPARISON OF RSD AND LOT-SIZES FOR WASHED ILLINOIS COALS

Lot-Size Interval (Tons)		RSD of Lbs S/MMBtu (%)	Number of Observations
Mid-Point	Range		
1,345	1,000-1,500	28.60	994
1,715	1,500-2,000	14.52	264
2,143	2,000-2,500	14.29	187
2,870	2,500-3,500	21.17	158
4,220	3,500-5,000	32.84	274
6,060	5,000-7,000	24.17	436
8,224	7,000-9,000	37.74	1,040
9,812	9,000-12,000	24.81	1,203

Source: Schedule J-1.

5.2 Illinois Seam No. 6 (484) and Seams No. 5 and No. 6 (900)

In Producing District 10, separate analyses were performed for the Illinois No. 6 seam and a composite of the No. 5 and No. 6 seams. Data for the composite of the No. 5 and No. 6 seams were obtained from mines engaged in multiple seam operations producing a mixed product.

Analysis of the data for the No. 6 seam indicated an average of 2.67 lbs S/MMBtu with an RSD of 34.23 percent. The results of the lot-size analyses for this seam, which are set out in Table J-6, did not demonstrate an inverse relationship between RSD and lot-size. In fact, the larger lot-sizes exhibited substantially greater RSD's than the smaller lot-sizes. An examination of the data indicated that the intervals exhibiting the larger RSD's also had relatively lower means for the lbs S/MMBtu.

TABLE J-6

COMPARISON OF RSD AND LOT-SIZES FOR ILLINOIS SEAM NO. 6

Lot-Size Interval (Tons)		Lbs S/MMBtu		Number of
Mid-Point	Range	Average	RSD (%)	Observations
1,388	1,000-2,000	2.85	28.76	993
2,354	2,000-3,000	2.89	13.57	172
3,971	3,000-5,000	2.93	16.61	72
5,897	5,000-6,200	3.33	14.31	186
7,262	6,200-7,800	2.20	43.08	155
8,369	7,800-9,000	2.17	45.62	461
9,581	9,000-10,000	2.40	45.28	204

Source: Schedule J-1.

The data sets for the Illinois No. 6 seam were analyzed a second time, based on the average sulfur content of the coals. Two categories of data sets were analyzed: (1) average sulfur content less than two percent, and (2) average sulfur content greater than two percent. This analysis provided an RSD of 40.8 percent for the data sets with an average sulfur content of less than two percent and an RSD of 13.3 percent for the data sets with an average sulfur content of greater than two percent.

This analysis of RSD versus average sulfur content was extended to other data sets in the Mid-Continent producing area (Illinois, Indiana, and western Kentucky). The results of this analysis are discussed in Section 13.0 of this Appendix.

The aggregate analysis for the composite of the Illinois No. 5 and No. 6 seams yielded an average of 2.75 lbs S/MMBtu and an RSD of 18.2 percent. A comparison of the composite with the Illinois No. 6 seam reveals that the composite has a slightly higher average lbs S/MMBtu but a significantly lower RSD as set out in Table J-7.

TABLE J-7

COMPARISON OF AVERAGE LBS S/MMBTU AND RSD FOR ILLINOIS SEAM NO. 6 AND A COMPOSITE OF ILLINOIS SEAMS NO. 5 AND NO. 6

	Lbs S/MMBtu	
Seam	Average	RSD (%)
Illinois No. 6	2.67	34.2
Illinois No. 5 and No. 6	2.75	18.2

Source: Schedule J-1.

As in the case of the Illinois No. 6 seam, the composite of the No. 5 and No. 6 seams failed to demonstrate an inverse relationship between RSD and lot-size.

5.0 Producing District 11 (Indiana)

Aggregate analyses for Producing District 11 coals were examined only with respect to sulfur contents. As in the case of the Illinois No. 6 seam, the Indiana data sets were individually analyzed for average sulfur contents less than and greater than two percent.

Data sets with an average sulfur content less than two percent had an average of 1.5 lbs S/MMBtu with an RSD of 35 percent, while data sets with sulfur contents greater than two percent exhibited an average of 3.25 lbs S/MMBtu with an RSD of 16 percent. Both cases failed to demonstrate an inverse relationship between RSD and lot-size.

6.0 Producing district 12 (Iowa)

The data available for Producing District 12 limited the analyses to an aggregate of all coals and the Lucas County No. 5 seam (517). The average lbs S/MMBtu for all coals was 3.88 with an RSD of 32.5 percent. Data for the Lucas County No. 5 seam indicated 3.35 lbs S/MMBtu and an RSD of 22.6 percent. In both cases insufficient data precluded a lot-size analysis.

7.0 Producing District 15 (Kansas and Missouri)

In Producing District 15 only data for the Bevier seam (492) were analyzed. Raw coals from this seam exhibited an average of 4.29 lbs S/MMBtu with an RSD of 25.8 percent, while the washed coals contained 3.38 lbs S/MMBtu with an RSD of 8.5 percent. The available data did not permit a lot-size analysis.

8.0 Producing District 17 (Colorado)

An aggregate analysis of all coals in Producing District 17 indicated an average of 0.51 lbs S/MMBtu with an RSD of 21.9 percent. In the interval analysis of these coals, RSD's from 11.7 to 25.8 percent were obtained. However, an inverse relationship between RSD and lot-size was not observed.

9.0 Producing District 18 (Arizona and New Mexico)

9.1 All Coals

Data for Producing District 18 consisted of three data sets representing two mines. An aggregate analysis of these data yielded an average of 0.45 lbs S/MMBtu with an RSD of 44.1 percent.

9.2 Automatic ASTM Samples

Within these data, the samples that were collected by automatic ASTM samplers were separately analyzed. These samples indicated an average of 0.43 lbs S/MMBtu with an RSD of 35.7 percent. An analysis of the automatic, ASTM samples by lot-size indicated RSD's ranging from 12.6 to 50.2 percent but failed to demonstrate an inverse relationship between RSD and lot-size.

9.3 Automatic ASTM Samples and Analyses, by Laboratory

A special analysis of Producing District 18 coals focused on the automatic, ASTM samples collected at the Navajo Power Plant. These samples were analyzed by two different laboratories -- Salt River Project (SRP) and Commercial Testing and Engineering (CTE). The results of the analysis of these data are set out in Table J-8. Sufficient data were not available to perform a lot-size analysis.

TABLE J-8

COMPARISON OF NAVAJO POWER PLANT AUTOMATIC, ASTM ANALYSES PERFORMED BY DIFFERENT LABORATORIES

Laboratory	Lbs S/MMBtu Average	Lbs S/MMBtu RSD (%)	Number of Observations
SRP	0.37	14.23	114
CTE	0.40	15.77	89

Source: Schedule J-1.

10.0 Producing District 19 (Wyoming)

10.1 All Coals

The aggregate analysis of all coals for Producing District 19 indicated an average of 0.51 lbs S/MMBtu with an RSD of 37.7 percent. A lot-size analysis of these data failed to indicate an inverse relationship between RSD and lot-size.

10.2 Comparison of All Coals to Individual Mines

Set out in Table J-9 is a comparison of all Producing District 19 coals to the individual mines from which the coal analyses were obtained. From Table J-9 it can be seen that there is substantial variation in the average lbs S/MMBtu and the RSD among the various mines. Although the average lbs S/MMBtu for Producing District 19 was found to be 0.51, individual mines exhibit a range from 0.38 to 0.96 lbs S/MMBtu. A similar comparison of the RSD's shows 37.7 percent for all coals in Producing District 19, while individual mines exhibit RSD's ranging from 9.9 to 45.4 percent.

TABLE J-9

COMPARISON OF PRODUCING DISTRICT 19 COALS TO INDIVIDUAL MINES WITHIN PRODUCING DISTRICT 19

Data Source	Lbs S/MMBtu		Number of Observations
	Average	RSD (%)	
Producing District 19, Total	0.51	37.7	3,526
Mine 10006	0.43	13.6	2,199
Mine 10007	0.47	9.9	312
Mine 10038	0.61	31.3	169
Mine 10039	0.81	45.4	58
Mine 10061	0.38	13.1	99
Mine 10076	0.57	34.1	25
Mine 10077	0.96	20.9	294

Source: Schedule J-1.

11.0 Producing District 21 (North Dakota)

An aggregate analysis of the lignite coals of North Dakota indicated an average of 1.18 lbs S/MMBtu with an RSD of 27.4 percent. The limited data available for a lot-size analysis failed to reveal an inverse relationship between RSD and lot-size.

12.0 Producing District 22 (Montana)

An aggregate analysis for Montana coals indicated an average of 0.65 lbs S/MMBtu with an RSD of 44 percent. Sufficient data were not available for a lot-size analysis.

13.0 Relationship of RSD of Lbs S/MMBtu to Average Sulfur Content in the Mid-Continent Producing Area

In the analysis of the Illinois No. 6 seam, it was observed that the data sets with lower average sulfur contents exhibited relatively higher RSD's for lbs S/MMBtu. This type of analysis was extended to include all data sets in the Mid-Continent producing area (Illinois, Indiana, and western Kentucky).

A basic assumption in this analysis is that coal sulfur contents and heat contents are independent. Numerous data sets were analyzed and the results of these analyses support this assumption.

The definition of RSD of lbs S/MMBtu is the standard deviation divided by the mean.

$$\text{RSD (lbs Sulfur/MMBtu)} = \frac{\sigma\text{(lbs S/MMBtu)}}{\bar{X}\text{ (lbs S/MMBtu)}}$$

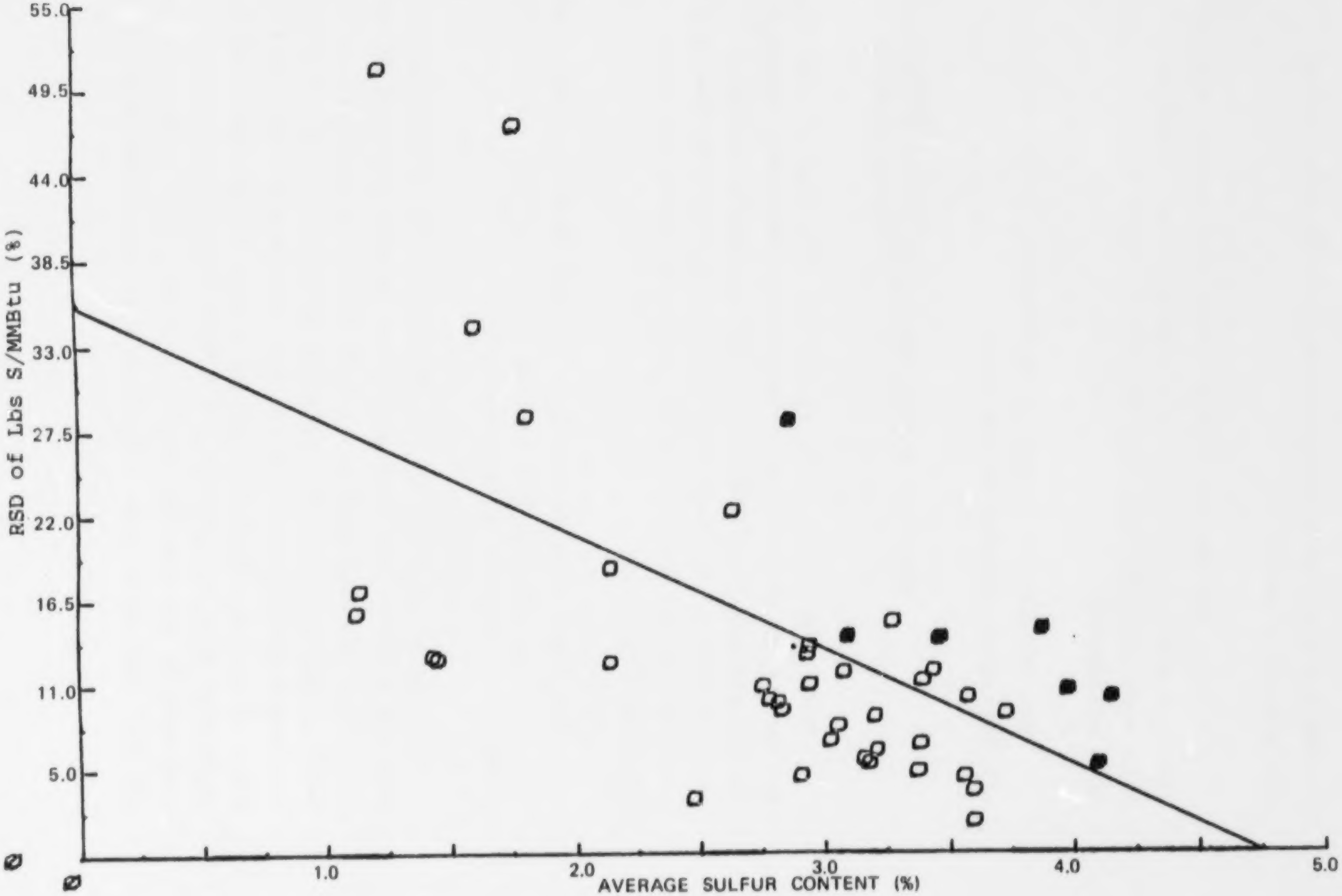
From this definition, it can be seen that if the standard deviation of coals were relatively constant, an increase in the average lbs S/MMBtu (or average sulfur content) would result in a lower RSD, while a decrease in the average lbs S/MMBtu (or average sulfur content) would result in a higher RSD.

The basic problem examined is the impact of the standard deviation on the RSD for various levels of the average lbs S/MMBtu (or average sulfur contents).

As shown on Figure J-1, the RSD of lbs S/MMBtu was plotted against the average sulfur content for each of 44 data sets representing coals from Illinois, Indiana, and western Kentucy. These coals are primarily washed coals; raw coals are designed by the solid points plotted on Figure J-1. A linear regression of the plots for the 44 data sets indicates that the RSD of lbs S/MMBtu decreases with an increase in the average sulfur content. The particular slope of the regression line indicates that the standard deviations

Figure J-1

COMPARISON OF RSD OF LBS. S/MMBtu AND AVERAGE SULFUR CONTENT FOR 44 DATA SETS IN ILLINOIS, INDIANA, AND WESTERN KENTUCKY



are relatively greater for the lower average sulfur content coals.

With respect to compliance with sulfur-limiting regulations, it is interesting to examine the consequences of the relationship in Figure J-1. First, for these Mid-Continent coals, there appears to be a trade-off between average sulfur content and variability among the mines. Given a specific sulfur emissions level, the selection of a lower sulfur source would require an assessment of the impact of increased variability. The attractiveness of a lower sulfur source may be mitigated to some extent due to intermittent periods of excess emissions, although the average level of emissions would be lower than those resulting from a higher sulfur content coal.

A second and related consequence of Figure J-1 concerns alternative strategies for obtaining compliance coals through washing. In general, comparisons of raw and washed coals have shown that within individual mines and coal seams washing reduces the average sulfur content and reduces the RSD of the lbs S/MMBtu. However, Figure J-1 indicates that the net benefits of the reduction in relative variability from coal washing may be less than expected as progressively lower sulfur coals are selected.

14.0 Summary of Analysis of Data on an Aggregate Basis

Although no defensible conclusions resulted from these aggregate analyses, it is possible to make some observations about relationships observed in the various Producing Districts.

On the basic question of the relationship of the RSD of lbs S/MMBtu to lot-size, the results were inconclusive.

However, it appears that, in general, as the level of aggregation increases from mine to seam to Producing District the data increasingly tend to exhibit an inverse relationship between RSD and lot-size.

Within the individual Producing Districts and coal seams analyzed, the washed coals consistently exhibited both a lower lbs S/MMBtu and a lower RSD than the raw coals. This tends to support the hypothesis that coal washing would reduce the average level of sulfur emissions as well as the relative variability of the emissions.

Based on the limited number of cases analyzed, it appears that substantial differences in RSD's can exist among seams within the same Producing District. Previous sections of this report indicated that substantial, inconsistent differences were observed among individual mines and among lot-sizes within mines, on a Producing District basis. These observations are reason for serious concern with respect to the extent of the relationship between RSD and lot-size and the existence of a simple relationship which can accurately generalize coal sulfur variability.

Finally, some of the data on an aggregate basis exhibited rather large RSD's, some in excess of 60 percent. RSD's of this magnitude could have a substantial impact on compliance with sulfur dioxide emissions regulations. This suggests that coal consumers subject to a given emissions limit with only marginally acceptable coals must selectively evaluate the alternative sources of supply and may find it necessary to exclude those sources exhibiting large variabilities.

Schedule J-1

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION
(INCLUDES LOT-SIZE ANALYSIS)

Data Description: USBM District, Seam, Preparation Lot-Sizes, etc.	Volume Sampled (tons)		Sulfur Content (Percent, As Received)		Heat Content (Btu/Lb., As Received)		Lbs. Sulfur/ MMBtu		Number of Analyses
	Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Dist. 01, All Coals	5677	88.51	2.16	24.61	11822	4.44	1.83	26.18	6619
Dist. 01, All Coals, 200-500 T	339	26.48	1.43	41.25	12276	5.67	1.17	42.49	289
Dist. 01, All Coals, 500-750 T	612	11.02	1.69	38.54	12038	6.25	1.40	38.98	257
Dist. 01, All Coals, 750-1000 T	861	8.70	1.89	31.25	12035	5.47	1.59	32.87	156
Dist. 01, All Coals, 1000-2000 T	1509	18.87	2.00	28.55	11903	4.84	1.69	29.45	698
Dist. 01, All Coals, 2000-3000 T	2506	11.52	2.28	25.38	11729	4.41	1.96	26.97	871
Dist. 01, All Coals, 4000-6000 T	4943	11.65	2.33	15.88	11637	3.38	2.00	17.07	956
Dist. 01, All Coals, 7000-9000 T	7832	7.24	2.32	11.46	11802	2.94	1.97	12.19	433
Dist. 01, All Coals, 9000-11000 T	10001	6.08	2.28	8.94	11827	2.68	1.93	9.26	287
Dist. 01, All Coals, 11000-13000 T	12036	4.79	2.29	9.15	11832	2.55	1.94	9.76	359
Dist. 01, All Coals, 13000-15000 T	14032	4.07	2.28	9.05	11872	2.12	1.92	9.82	428
Dist. 01, All Coals, 19000-23000 T	20088	3.99	2.22	9.77	11798	3.87	1.89	10.41	50
Dist. 01, Raw	6111	84.85	2.19	24.60	11804	4.37	1.86	26.22	5738
Dist. 01, Raw, 100-300 T	169	32.46	1.36	38.08	12400	5.46	1.10	39.19	349
Dist. 01, Raw, 300-500 T	397	14.45	1.52	40.93	12292	5.35	1.24	42.01	135
Dist. 01, Raw, 500-750 T	605	11.10	1.66	40.78	12000	6.53	1.37	41.19	213
Dist. 01, Raw, 1000-1200 T	1097	5.38	1.93	29.09					88
Dist. 01, Raw, 2000-3000 T			2.37	25.11	11682	4.31	2.04	26.68	666
Dist. 01, Raw, 3000-4000 T	3479	8.17	2.41	22.55	11481	4.37	2.11	24.20	625
Dist. 01, Raw, 4000-5000 T	4496	6.40	2.40	16.70	11553	3.12	2.09	17.81	427
Dist. 01, Raw, 5000-7000 T	5937	9.26	2.34	11.12	11653	2.65	2.01	11.90	690
Dist. 01, Raw, 7000-9000 T	7833	7.29	2.33	10.78	11777	2.75	1.98	11.44	411
Dist. 01, Raw, 9000-11000 T	9994	6.11	2.28	8.87	11823	2.62	1.93	9.28	281
Dist. 01, Raw, 11000-14000 T	12576	6.91	2.29	9.24	11850	2.43	1.93	9.88	561
Dist. 01, Washed	2851	81.25	1.96	22.12	11942	4.73	1.65	22.78	881
Dist. 01, Washed, 100-300 T	266	15.92	1.53	22.29	12109	5.74	1.26	20.38	25
Dist. 01, Washed, 300-500 T	415	14.88	1.58	39.75	11986	.28	1.32	39.59	35
Dist. 01, Washed, 500-1000 T	757	16.28	1.82	28.23	12178	5.34	1.50	29.83	92
Dist. 01, Washed, 1000-1500 T	1228	11.74	1.97	22.64	11916	4.57	1.66	23.30	89
Dist. 01, Washed, 1500-2000 T	1733	8.85	2.08	15.39	11816	3.69	1.76	14.93	75
Dist. 01, Washed, 2000-3000 T	2506	11.35	1.99	20.14	11883	4.45	1.68	20.67	205
Dist. 01, Washed, 3000-4000 T	3424	8.63	2.10	15.05	11758	3.36	1.79	15.71	126
Dist. 01, Washed, 4000-6000 T	4799	10.75	2.02	17.53	11873	4.68	1.71	18.05	155
Dist. 01, Washed, 6000-9000 T	7200	9.89	2.10	17.57	12344	3.25	1.71	17.56	46
Dist. 01, Seam 071, Raw	4134	46.39	2.44	18.31	11583	3.26	2.12	19.78	2312
Dist. 01, Seam 071, Washed	3034	57.39	2.07	11.88	11614	2.54	1.79	12.22	488
Dist. 01, Seam 071, Washed, 0-500 T	401	12.36	2.19	13.32	11530	2.31	1.90	13.14	8
Dist. 01. Seam 071, 500-1000 T	756	18.71	2.07	12.60	11467	3.16	1.81	12.45	25
Dist. 01, Seam 071, 1000-2000 T	1546	17.20	2.09	10.64	11637	2.57	1.80	11.01	107
Dist. 01, Seam 071, 3000-4000 T	3424	8.70	2.11	11.50	11658	2.45	1.82	12.16	101
Dist. 01, Seam 071, 4000-5500 T	4735	8.62	2.04	13.42	11607	2.45	1.76	13.73	101
Dist. 04, Raw/Delivered	1458	57.84	3.92	22.60	11015	5.47	3.58	24.81	6362
Dist. 04, Raw/Delvd., 200-500 T	376	22.63	4.02	24.29	10994	7.18	3.70	29.47	227
Dist. 04, Raw/Delvd., 500-750 T	814	13.27	3.94	20.96	11027	5.94	3.61	24.49	275

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION
(INCLUDES LOT-SIZE ANALYSIS)

Data Description: USRM District, Seam, Preparation Lot-Sizes, etc. (1)	Volume Sampled (tons) Average (2)	Volume Sampled (tons) RSD(%) (3)	Sulfur Content (Percent, As Received) Average (4)	Sulfur Content (Percent, As Received) RSD(%) (5)	Heat Content (Btu/Lb., As Received) Average (6)	Heat Content (Btu/Lb., As Received) RSD(%) (7)	Lbs. Sulfur/ MMBtu Average (8)	Lbs. Sulfur/ MMBtu RSD(%) (9)	Number of Analyses (10)
Dist. 04, Raw/Delvd., 750-1000 T	925	7.75	3.84	23.19	11062	5.39	3.50	26.78	701
Dist. 04, Raw/Delvd., 1000-1500 T	1257	12.46	3.80	21.96	11100	4.77	3.44	22.98	1479
Dist. 04, Raw/Delvd., 1500-2000 T	1667	8.23	3.95	21.23	10992	5.63	3.61	23.38	880
Dist. 04, Raw/Delvd., 2000-3000 T	2447	12.17	4.33	20.87	10616	6.48	4.10	22.61	341
Dist. 04, Raw/Delvd., 3000-4000 T	3419	8.40	4.49	20.83	10582	5.88	4.25	21.62	244
Dist. 04, Raw/Delvd., 4000-5000 T	4367	5.80	4.64	21.15	10675	5.64	4.37	22.43	148
Dist. 04, Raw/As Burned	3260	57.67	3.90	17.68	10809	5.16	3.62	19.00	2251
Dist. 04, Raw/As Burned, 200-500 T	361	23.77	3.92	17.04	10865	3.49	3.63	18.02	26
Dist. 04, Raw/As Burned, 500-1000 T	805	15.02	3.93	20.63	10860	4.51	3.64	22.72	124
Dist. 04, Raw/As Burned, 1000-1500 T	1266	11.21	3.85	20.08	10968	4.49	3.53	21.98	232
Dist. 04, Raw/As Burned, 1500-2500 T	2015	14.36	3.92	16.78	10941	4.41	3.60	18.77	492
Dist. 04, Raw/As Burned, 2500-3500 T	3049	9.43	3.87	17.69	10715	5.62	3.63	19.00	563
Dist. 04, Raw/As Burned, 3500-5500 T	4063	11.68	3.84	17.53	10748	5.25	3.60	17.95	467
Dist. 04, Raw/As Burned, 6000-8000 T	6974	6.75	3.99	15.70	10702	5.50	3.74	16.14	294
Dist. 04, Source 10175, Units 1,2 & 3 As Burned	2578	45.14	4.29	14.07	10634	3.36	4.04	15.34	614
Dist. 04, Source 10175, Unit 4, As Burned	6246	26.38	4.04	15.45	10693	5.63	3.79	16.10	402
Dist. 04, Source 10175, Unit 4, As Burned									
2500-5000 T	3918	13.95	3.92	15.57	10660	6.74	3.68	14.25	46
5000-6000 T	5512	4.46	4.27	18.24	10695	5.62	4.01	19.82	39
6000-7000 T	6594	4.22	3.99	14.71	10728	5.33	3.73	15.12	141
7000-8000 T	7361	3.48	4.03	14.21	10669	5.61	3.79	14.85	146
Dist. 04, Source 10175, Unit 5, As Burned	3382	20.23	3.74	15.25	10616	6.10	3.53	14.34	410
Dist. 04, Source 10175, Unit 5, As Burned									
2000-3000 T	2564	11.60	3.69	11.92	10569	6.67	3.50	11.11	69
3000-3500 T	3263	4.42	3.71	41.28	10618	6.37	3.50	13.05	151
3500-4000 T	3730	3.66	3.74	16.77	10618	5.76	3.52	15.53	127
4000-4500 T	4157	2.99	3.89	14.29	10697	5.18	3.64	13.92	49
Dist. 04, Source 10175, Unit 6, As Burned	2868	40.78	3.87	16.66	10632	6.26	3.65	16.12	139
Dist. 04, Source 10175, Unit 6, As Burned									
1300-2000 T	1628	12.08	3.67	24.90	10598	4.81	3.46	23.87	20
2000-3000 T	2593	10.52	3.90	13.89	10459	6.84	3.73	13.89	38
3000-4000 T	3493	8.45	3.91	15.42	10680	6.13	3.67	15.36	54
Dist. 04, Seam 036 Raw									
600-1000 T	821	14.48	3.70	21.23	11098	4.75	3.33	21.96	15
1000-1300 T	1170	7.32	3.88	17.20	11018	5.50	3.51	16.07	42
1300-1600 T	1448	5.28	3.59	23.48	11038	5.48	3.25	23.33	78
1600-2100 T	1782	6.82	3.65	18.79	11108	4.42	3.28	17.77	47
Dist. 04, Seam 080, Raw									
300-600 T	456	17.49	4.18	28.05	10626	8.67	4.00	33.68	90
600-900 T	763	12.00	4.26	26.22	10553	7.49	4.10	31.57	120

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION
(INCLUDES LOT-SIZE ANALYSIS)

Data Description: USDM District, Seam, Preparation Lot-Sizes, etc. (1)	Volume Sampled (tons) Average (2)	Volume Sampled (tons) RSD(%) (3)	Sulfur Content (Percent, As Received) Average (4)	Sulfur Content (Percent, As Received) RSD(%) (5)	Heat Content (Btu/Lb., As Received) Average (6)	Heat Content (Btu/Lb., As Received) RSD(%) (7)	Lbs. Sulfur/ MMBtu Average (8)	Lbs. Sulfur/ MMBtu RSD(%) (9)	Number of Analyses (10)
Dist. 04, Seam 080, Raw									
1000-1300 T	1115	9.11	3.41	28.39	10973	5.07	3.13	31.96	426
1300-1600 T	1425	5.68	3.84	23.91	10910	5.88	3.54	26.07	394
1600-1900 T	1745	5.16	4.00	24.67	10718	6.58	3.76	28.40	198
2000-2300 T	2137	4.12	4.39	20.60	10431	7.25	4.24	22.61	101
2600-3000 T	2796	4.30	4.61	17.61	10613	6.10	4.35	18.21	89
3000-4000 T	3441	8.23	4.56	19.76	10570	5.94	4.33	20.56	229
4000-5000 T	4357	5.78	4.72	19.68	10679	5.72	4.34	21.09	142
Dist. 04, Seam 084 Raw									
250-600 T	440	23.23	4.05	20.01	11064	4.98	3.66	20.74	29
900-1100 T	1015	5.36	3.86	19.90	11188	4.07	3.45	20.23	105
1100-1300 T	1208	4.92	3.99	20.49	11175	3.89	3.57	20.44	188
1300-1500 T	1407	3.72	4.16	18.92	11235	4.21	3.70	19.87	517
1500-1800 T	1582	5.42	4.33	16.97	11348	4.17	3.82	17.19	91
2000-3000 T	2470	10.92	4.15	19.56	11159	4.74	3.71	18.28	22
Dist. 08, All Coals	5732	176.60	.99	46.08	12022	5.81	0.83	47.71	3299
Dist. 08, All Coals, 200-500 T	339	26.40	.89	33.91	12151	5.03	0.74	34.54	140
Dist. 08, All Coals, 500-1000 T	751	20.40	.96	38.88	12149	4.98	0.80	39.71	156
Dist. 08, All Coals, 1000-1500 T	1351	13.41	.94	46.14	12197	5.55	0.78	47.64	377
Dist. 08, All Coals, 1500-2000 T	1637	9.88	.92	39.43	12263	5.77	0.75	39.30	389
Dist. 08, All Coals, 2000-3000 T	2487	10.72	.96	38.91	12412	8.06	0.78	40.19	316
Dist. 08, All Coals, 3000-4000 T	3571	8.74	.91	46.24	11891	5.24	0.77	46.90	205
Dist. 08, All Coals, 4000-5000 T	4392	6.14	.89	40.31	11958	6.13	0.74	40.80	309
Dist. 08, All Coals, 5000-7500 T	6140	8.54	1.08	55.38	11771	4.54	0.92	56.08	996
Dist. 08, All Coals, 7500-10000 T	8526	7.56	.97	34.55	12412	4.94	0.78	35.59	228
Dist. 08, All Coals, 15000-18000 T	16583	6.14	1.08	16.70	11666	4.64	0.93	19.38	27
Dist. 08, All Coals, 25000-35000 T	29898	8.61	.98	13.70	12136	3.42	0.81	14.63	32
Dist. 08, Raw	4095	58.68	1.21	47.12	12288	7.06	1.00	50.97	1211
Dist. 08, Raw, 900-1200 T	1085	4.10	1.15	58.55	12392	6.30	0.95	63.29	65
Dist. 08, Raw, 1200-1500 T	1489	2.56	.91	44.88	12275	5.74	0.75	45.41	204
Dist. 08, Raw, 1500-2000 T	1601	9.51	.92	42.25	12434	5.58	0.75	42.60	284
Dist. 08, Raw, 2000-3000 T	2450	9.67	1.03	41.00	13023	7.36	0.80	45.35	163
Dist. 08, Raw, 3000-4000 T	3546	9.10	1.27	40.72	12291	7.07	1.05	45.40	48
Dist. 08, Raw, 4000-6000 T	5307	14.46	1.57	46.18	11897	7.15	1.34	48.47	303
Dist. 08, Raw, 6000-8000 T	6474	7.73	1.60	41.62	11851	5.49	1.37	44.08	316
Dist. 08, Washed	5140	65.61	.78	37.74	12039	5.50	0.65	38.49	346
Dist. 08, Washed, 1000-2000 T	1535	19.89	.78	30.94	11675	4.41	0.67	29.39	65
Dist. 08, Washed, 2000-3000 T	2550	10.96	.72	23 44	11567	4.68	0.62	23.03	53
Dist. 08, Washed, 3000-6000 T	4295	23.17	.76	37.61	11983	5.01	0.64	36 57	78
Dist. 08, Washed, 6000-12000 T	8936	15.75	.83	43.60	12452	4.60	0.67	47.06	136
Dist. 08, Seam 151	2253	65.37	.90	47.62	11692	5.49	0.77	47.32	188
Dist. 08, Seam 151, 900-1400 T	1110	4.13	1.05	57.44	12052	5.58	0.88	60.73	64
Dist. 08, Seam 151, 1400-2000 T	1737	11.20	.86	34.04	11554	3.78	0.74	30.90	32
Dist. 08, Seam 151, 2000-4000 T	2812	16.97	.79	26.17	11416	4.05	0.69	22.89	62

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION
(INCLUDES LOT-SIZE ANALYSIS)

Data Description: USBM District, Seam, Preparation Lot-Sizes, etc. (1)	Volume Sampled (tons) Average (2)	Volume Sampled (tons) RSD(%) (3)	Sulfur Content (Percent, As Received) Average (4)	Sulfur Content (Percent, As Received) RSD(%) (5)	Heat Content (Btu/Lb., As Received) Average (6)	Heat Content (Btu/Lb., As Received) RSD(%) (7)	Lbs. Sulfur/ MMBtu Average (8)	Lbs. Sulfur/ MMBtu RSD(%) (9)	Number of Analyses (10)
Dist. 08, Seam 157	4273	59.87	1.03	30.48	13073	4.57	0.79	31.33	392
Dist. 08, Seam 157, 1000-2000 T	1601	21.50	1.02	41.43	13404	4.15	0.77	42.51	52
Dist. 08, Seam 157, 2400-2900 T	2461	3.30	.99	41.27	13376	4.31	0.74	42.34	95
Dist. 08, Seam 157, 4000-4900 T	4221	3.01	.98	17.18	13008	2.80	0.76	17.51	69
Dist. 08, Seam 157, 7000-9000 T	8170	4.21	1.12	16.90	12678	3.86	0.88	16.70	106
Dist. 09, All Coals	3757	95.64	3.11	25.49	10995	4.14	2.84	26.77	3905
Dist. 09, All Coals, 200-500 T	432	22.06	3.47	14.96	10951	4.34	3.19	17.03	17
Dist. 09, All Coals, 500-1000 T	697	29.15	3.74	16.05	10753	4.43	3.50	18.09	26
Dist. 09, All Coals, 1000-1500 T	1390	6.10	2.63	26.60	11054	5.20	2.40	29.35	1553
Dist. 09, All Coals, 1500-2000 T	1562	5.54	2.48	27.53	10974	3.40	2.26	29.00	516
Dist. 09, All Coals, 2000-3000 T	2521	13.22	3.62	14.13	10969	3.60	3.31	15.55	58
Dist. 09, All Coals, 3000-4000 T	3451	6.05	3.74	7.85	10902	2.79	3.44	8.42	143
Dist. 09, All Coals, 4000-6000 T	4436	10.88	3.61	11.95	10806	3.02	3.35	11.61	660
Dist. 09, All Coals, 6000-8000 T	6924	3.89	3.81	8.01	11009	2.36	3.47	9.01	865
Dist. 09, Raw	4662	52.79	3.65	16.47	11113	4.10	3.31	18.74	1783
Dist. 09, Raw, 200-500 T	431	22.78	3.59	6.14	10897	4.01	3.31	8.92	16
Dist. 09, Raw, 500-1000 T	687	29.15	3.84	9.77	10726	4.34	3.59	12.31	25
Dist. 09, Raw, 1000-1500 T	1359	8.28	3.00	26.30	11539	4.85	2.63	30.39	412
Dist. 09, Raw, 1500-2000 T	1634	9.79	3.69	16.47	10956	6.48	3.41	21.53	81
Dist. 09, Raw, 2000-3000 T	2479	13.75	3.75	9.50	10980	3.68	3.43	11.78	46
Dist. 09, Raw, 3000-4000	3454	6.00	3.76	7.44	10895	2.72	3.46	7.88	140
Dist. 09, Raw, 4000-5000 T	4516	4.07	4.04	8.10	10993	2.77	3.68	8.43	235
Dist. 09, Raw, 5000-6000 T	5508	6.65	3.85	7.87	10984	2.47	3.52	9.16	45
Dist. 09, Raw, 6000-8000 T	6921	3.88	3.83	7.67	11012	2.27	3.48	8.73	840
Dist. 09, Washed	3255	136.11	2.55	23.06	10984	3.30	2.33	24.21	1834
Dist. 09, Washed, 0-1000 T	706	36.12	1.47	7.12	11619	1.66	1.27	5.81	2
Dist. 09, Washed, 1000-1500 T	1422	3.82	2.22	15.57	11056	2.94	2.01	15.05	861
Dist. 09, Washed, 1500-2000 T	1549	3.55	2.23	15.92	10984	2.40	2.03	14.61	427
Dist. 09, Washed, 2000-4000 T	2796	13.09	3.11	20.09	10932	3.02	2.85	20.48	14
Dist. 09, Washed, 4000-6000 T	4281	10.47	3.35	7.54	10685	2.66	3.15	9.09	398
Dist. 09, Washed, 6000-10000 T	7615	12.98	3.22	6.62	11048	4.41	2.93	9.83	37
Dist. 09, Washed, 10000-20000 T	14894	18.85	3.09	5.44	11531	3.10	2.69	6.64	59
Dist. 09, Washed, 20000-30000 R	24047	10.37	3.08	4.55	11473	2.32	2.69	5.57	49
Dist. 09, Seam 484, Raw	4216	16.23	3.98	7.96	10940	2.74	3.64	7.66	296
Dist. 09, Seam 484, Raw, 3300-3600 T	3420	00.00	3.69	4.70	10820	2.57	3.42	5.29	84
Dist. 09, Seam 484, Raw, 4400-4800 T	4544	01.41	4.00	7.48	10973	2.83	3.72	7.37	168
Dist. 09, Seam 484 Washed	4394	22.23	3.34	8.12	10688	2.66	3.14	9.58	431
Dist. 09, Seam 484, Washed, 4200-4800 T	4512	1.58	3.01	7.40	10961	3.31	2.75	6.71	6
Dist. 09, Seam 484, Washed, 5500-6500 T	5992	5.35	3.12	6.74	11123	2.62	2.81	5.56	3

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION
(INCLUDES LOT-SIZE ANALYSIS)

Data Description: USBM District, Seam, Preparation Lot-Sizes, etc. (1)	Volume Sampled (tons) Average (2)	Volume Sampled (tons) RSD(%) (3)	Sulfur Content (Percent, As Received) Average (4)	Sulfur Content (Percent, As Received) RSD(%) (5)	Heat Content (Btu/Lb., As Received) Average (6)	Heat Content (Btu/Lb., As Received) RSD(%) (7)	Lbs. Sulfur/ MMBtu Average (8)	Lbs. Sulfur/ MMBtu RSD(%) (9)	Number of Analyses (10)
Dist. 10, All Coals	5921	58.74	2.84	28.51	10788	4.11	2.65	29.46	4516
Dist. 10, All Coals, 500-1000 T	843	16.88	3.15	18.12	10806	1.97	2.93	20.25	5
Dist. 10, All Coals, 1000-1500 T	1345	6.44	3.06	26.47	10842	5.27	2.86	28.60	994
Dist. 10, All Coals, 1500-2000 T	1715	11.57	3.19	15.11	10720	3.25	2.98	14.52	264
Dist. 10, All Coals, 2000-2500 T	2143	7.17	2.90	13.04	10652	3.71	2.73	14.29	187
Dist. 10, All Coals, 2500-3500 T	2870	8.38	2.95	20.22	10565	3.13	2.81	21.17	158
Dist. 10, All Coals, 3500-5000 T	4220	9.12	2.49	32.15	10746	3.66	2.32	32.84	274
Dist. 10, All Coals, 5000-7000 T	6060	8.50	3.10	23.32	10717	3.35	2.91	24.17	436
Dist. 10, All Coals, 7000-9000 T	8224	6.27	2.45	37.14	10842	4.20	2.80	37.74	1040
Dist. 10, All Coals, 9000-12000 T	9812	5.01	2.87	24.58	10781	3.39	2.66	24.81	1203
Dist. 10, Seam 900	8340	31.99	2.98	18.51	10834	3.11	2.75	18.16	1588
Dist. 10, Seam 900, 1500-2200 T	1563	2.79	3.48	12.38	10967	3.36	3.18	11.88	70
Dist. 10, Seam 900, 7000-8000 T	7470	4.09	3.07	12.82	10908	2.82	2.82	12.44	109
Dist. 10, Seam 900, 8000-9000 T	8633	3.18	2.75	20.59	10760	3.09	2.55	19.77	238
Dist. 10, Seam 900, 9000-10000 T	9575	3.30	2.89	20.89	10752	3.21	2.69	21.12	647
Dist. 10, Seam 900, 10000-12000 T	10289	3.00	3.20	13.48	10826	3.14	2.97	14.84	418
Dist. 10, Seam 484, Average Sulfur <2%	5932	58.02	1.40	39.58	11219	5.84	1.25	40.81	544
Dist. 10, Seam 484, Average Sulfur <2%, 1000-1500 T	1317	7.59	1.47	19.99	11812	3.40	1.25	21.86	184
7000-8000 T	7498	4.14	1.41	50.30	11142	4.24	1.27	49.26	71
8000-9000 T	8527	3.15	1.31	46.07	10923	4.65	1.21	47.25	172
9000-10000 T	9341	2.35	1.32	47.72	10666	5.42	1.25	48.33	81
Dist. 10, Seam 484, Average Sulfur >2%	4120	75.55	3.30	11.99	10627	3.28	3.12	13.26	1946
Dist. 10, Seam 484, Average Sulfur >2%, 6000-7000 T	6149	5.18	3.57	8.61	10586	1.76	3.38	9.16	182
Dist. 11, All Coals	5836	53.70	3.12	29.97	10853	2.70	2.88	30.45	1188
Dist. 11, Average Sulfur <2%,	7426	35.41	1.65	34.45	10836	1.99	1.53	34.98	257
Dist. 11, Average Sulfur <2%, 6000-7000 T	6508	4.17	1.72	38.09	10799	1.92	1.60	38.49	28
Dist. 11, Average Sulfur <2%, 7000-8000 T	7575	3.78	1.68	32.42	10809	1.62	1.55	32.03	50
Dist. 11, Average Sulfur <2%, 8000-9000 T	8445	3.67	1.65	34.32	10805	1.74	1.54	35.08	59
Dist. 11, Average Sulfur <2%, 9000-10000 T	9398	3.04	1.76	33.39	10847	1.71	1.62	33.71	46
Dist. 11, Average Sulfur >2%	5397	57.81	3.52	14.81	10858	2.86	3.25	15.74	911
Dist. 11, Average Sulfur >2%, 200-500 T	366	26.72	3.36	18.73	10939	2.54	3.08	20.07	21
Dist. 11, Average Sulfur >2%, 500-1000 T	783	16.62	3.41	15.28	10752	3.29	3.18	16.65	41
Dist. 11, Average Sulfur >2%, 1000-1500 T	1285	11.06	3.35	10.62	10826	2.65	3.10	11.17	42
Dist. 11, Average Sulfur >2%, 1500-2000 T	1753	8.32	3.47	16.73	10818	3.02	3.22	18.20	51
Dist. 11, Average Sulfur >2%, 2000-3000 T	2443	11.34	3.45	12.82	10755	3.41	3.22	14.63	81
Dist. 11, Average Sulfur >2%, 3000-5000 T	4156	14.53	3.53	15.23	10858	2.80	3.26	16.68	185
Dist. 11, Average Sulfur >2%, 5000-7000 T	6096	9.39	3.64	14.34	10874	2.81	3.35	14.64	227
Dist. 11, Average Sulfur >2%, 7000-11000 T	8625	12.32	3.50	14.78	10802	2.68	3.22	15.59	230

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION (INCLUDES LOT-SIZE ANALYSIS)

Data Description: USBM District, Seam, Preparation Lot-Sizes, etc. (1)	Volume Sampled (tons) Average (2)	Volume Sampled (tons) RSD(%) (3)	Sulfur Content (Percent, As Received) Average (4)	Sulfur Content (Percent, As Received) RSD(%) (5)	Heat Content (Btu/Lb., As Received) Average (6)	Heat Content (Btu/Lb., As Received) RSD(%) (7)	Lbs. Sulfur/ MMBtu Average (8)	Lbs. Sulfur/ MMBtu RSD(%) (9)	Number of Analyses (10)
Dist. 12, All Coals	3965	105.86	3.62	32.04	9346	5.13	3.08	32.47	203
Dist. 12, All Coals, 200-1000 T	638	35.47	3.41	29.83	9125	5.97	3.74	30.11	36
Dist. 12, All Coals, 1000-1500 T	1322	11.34	3.22	22.31	9390	3.69	3.43	22.47	41
Dist. 12, All Coals, 1500-2000 T	1708	6.35	3.43	27.30	9410	2.92	3.65	27.19	27
Dist. 12, Seam 517, Raw	4384	105.29	3.10	21.36	9302	4.71	3.35	22.61	150
Dist. 12, Seam 517, Raw, 200-500 T	353	23.34	2.27	17.15	8674	5.21	2.64	19.88	9
Dist. 12, Seam 517, Raw, 500-1000 T	759	18.53	3.23	27.72	9105	6.29	3.57	31.55	15
Dist. 12, Seam 517, Raw, 1000-2000 T	1482	15.84	3.14	20.42	9377	3.53	3.35	20.59	57
Dist. 15, Seam 492, Raw	3011	97.18	4.11	18.95	9770	8.23	4.29	25.78	92
Dist. 15, Seam 492, Raw, 1000-2000 T	1503	18.47	3.20	15.41	10374	7.22	3.13	21.55	12
Dist. 15, Seam 492, Raw, 45000-75000 T	58714	11.87	4.56	14.27	9515	7.63	4.87	21.13	35
Dist. 15, Seam 492, Washed	753	60.65	3.63	7.29	10771	2.61	3.38	8.49	33
Dist. 17, All Coals	3085	56.93	0.54	22.70	10546	4.35	0.51	21.88	495
Dist. 17, All Coals, 1300-1800 T	1576	3.92	0.60	12.15	10733	3.14	0.56	11.75	182
Dist. 17, All Coals, 2000-3000 T	2636	11.07	0.56	20.17	10718	4.22	0.53	19.04	72
Dist. 17, All Coals, 3000-4000 T	3108	4.26	0.48	26.48	10473	3.52	0.46	25.79	121
Dist. 17, All Coals, 6000-6900 T	6166	3.59	0.45	20.88	10124	3.91	0.45	22.56	91
Dist. 18, All Coals	10776	37.82	0.47	43.16	10485	4.34	0.45	44.06	299
Dist. 1[illegible] Auto., ASTM Samples	11105	38.11	0.44	34.67	10503	3.92	0.43	35.65	261
Dist. 18, Auto., ASTM Samples, 5000-7000 T	6182	4.25	0.43	11.78	10203	2.94	0.42	12.61	47
Dist. 18, Auto., ASTM Samples, 8000-10000 T	8872	5.05	0.63	49.81	10571	4.25	0.60	50.20	41
Dist. 18, Auto., ASTM Samples, 10000-13000 T	11462	6.73	0.39	10.85	10592	2.86	0.37	12.75	31
Dist. 18, Auto., ASTM Samples, 13000-16000 T	14915	5.18	0.40	11.93	10611	3.24	0.38	15.33	111
Dist. 18, Mine 10071 LAB - CTE	12200	34.43	0.42	11.65	10577	4.00	0.40	15.77	89
Dist. 18, Mine 10071, LAB - CTE 13000-17000 T	15017	5.27	0.42	11.19	10626	3.52	0.40	15.03	54
Dist. 18, Mine 10071 LAB- SRF	12318	31.88	0.39	10.95	10571	3.11	0.37	14.23	114

SAMPLE STATISTICS OF COAL CHARACTERISTICS ANALYZED BY VARIOUS LEVELS OF AGGREGATION
(INCLUDES LOT-SIZE ANALYSIS)

Data Description: USBM District, Seam, Preparation Lot-Sizes, etc.	Volume Sampled (tons)		Sulfur Content (Percent, As Received)		Heat Content (Btu/Lb., As Received)		Lbs. Sulfur/ MMBtu		Number of Analyses
	Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	Average	RSD(%)	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Dist. 19, All Coals	9766	25.78	0.45	47.30	8739	7.92	0.51	37.69	3526
Dist. 19, All Coals, 1000-2000 T	1605	17.39	1.01	19.82	10298	3.68	0.99	22.13	38
Dist. 19, All Coals, 2000-3000 T	2745	11.65	0.92	18.15	10302	3.72	0.89	18.10	157
Dist. 19, All Coals, 8000-10000 T	9737	4.36	0.58	43.88	9208	8.33	0.61	37.35	813
Dist. 19, All Coals, 10000-11000 T	10466	3.18	0.41	31.07	8647	6.20	0.48	25.60	2023
Dist. 19, All Coals, 11000-12000 T	11308	2.21	0.36	13.06	8391	2.24	0.43	12.45	738
Dist. 19, Seam 951	10222	19.90	0.36	15.09	8393	1.47	0.43	14.79	2224
Dist. 19, 10006 (All Analyses)	10234	19.94	0.36	14.01	8394	1.43	0.43	13.62	2199
Dist. 19, Mine 10007 (All Analyses)	10794	11.82	0.39	10.23	8301	1.84	0.47	9.88	312
Dist. 19, Mine 10038 (All Analyses)	9784	5.54	0.60	33.63	9852	3.74	0.61	31.33	169
Dist. 19, Mine 10039 (All Analyses)	9544	7.00	0.85	45.22	10626	3.10	0.81	45.44	58
Dist. 19, Mine 10061 (All Analyses)	10594	4.16	0.33	13.09	8781	1.51	0.38	13.08	99
Dist. 19, Mine 10076 (All Analyses)	9179	10.25	0.47	34.55	8266	2.98	0.57	34.06	25
Dist. 19, Mine 10077 (All Analyses)	4187	64.36	0.98	20.65	10271	3.13	0.96	20.93	294
Dist. 21, All Coals	13127	373.21	0.78	25.80	6684	7.07	1.18	27.37	2040
Dist. 21, All Coals, 0-30000 T	5345	74.06	0.79	25.16	6684	7.17	1.19	26.78	1986
Dist. 21, All Coals, 200000-400000 T	336277	11.23	0.55	28.83	6663	[illegible]	0.82	28.87	42
Dist. 22, All Coals	9243	9.13	0.58	39.65	9107	5.88	0.65	43 94	1196

APPENDIX K

ANALYSIS OF THE STATISTICAL DISTRIBUTIONS OF COAL CHARACTERISTICS

This Appendix provides a detailed discussion of the analyses performed to determine which type of frequency distribution-normal, inverted gamma, or lognormal-best fits the observed frequency distributions of actual coal data. Section 4.3 of this report discussed the methods used in this analysis and provided a summary of the results. The following discussion and tables provide the specific results obtained for the various analyses performed for both raw and washed coals.

1.0 Raw Coals

1.1 Raw Coals, Overall Goodness of Fit

The results of the chi-square goodness of fit test for raw coals are presented in Table K-1. "Best fit" in this table refers to the lowest chi-square value. The term "significant" means that the observed distribution was not significantly different from the assumed distribution. To illustrate, consider the analysis of heat content values for U.S. Bureau of Mines Producing District 1. For two of the data sets analyzed, the hypothesis that the data came from a normal distribution could not be rejected. Similarly, that hypothesis cannot be rejected for the inverted gamma distribution for two data sets, and for the lognormal distribution for one data set. The best fit in terms of the lowest chi-square value for Btu occurred once for the normal distribution and three times for the inverted gamma distribution. In this example, no data sets indicated a best fit for the lognormal distribution.

Sheet 1 of 2

TABLE K-1

CHI-SQUARE ANALYSIS, RAW COAL

U.S.B.M. Producing District	Variable		Normal	Inverted Gamma	Log-normal
1	Sulfur	(Best Fit)	4	-	-
		(Significant)	1	1	1
	Btu	(Best Fit)	1	3	-
		(Significant)	2	2	1
	Lbs S/MMBtu	(Best Fit)	2	-	2
		(Significant)	1	1	2
2	Sulfur	(Best Fit)	4	4	2
		(Significant)	3	3	3
	Btu	(Best Fit)	7	-	1
		(Significant)	5	5	3
	Lbs S/MMBtu	(Best Fit)	3	4	3
		(Significant)	2	4	1
8	Sulfur	(Best Fit)	-	2	1
		(Significant)	-	3	1
	Btu	(Best Fit)	-	1	-
		(Significant)	1	1	-
	Lbs S/MMBtu	(Best Fit)	-	2	1
		(Significant)	1	3	2
9	Sulfur	(Best Fit)	1	1	1
		(Significant)	1	1	1
	Btu	(Best Fit)	-	2	1
		(Significant)	1	1	1
	Lbs S/MMBtu	(Best Fit)	1	1	1
		(Significant)	1	1	1
10	Sulfur	(Best Fit)	2	2	1
		(Significant)	-	2	-
	Btu	(Best Fit)	2	1	-
		(Significant)	-	-	-
	Lbs S/MMBtu)	(Best Fit)	2	2	1
		(Significant)	-	1	-
11	Sulfur	(Best Fit)	1	2	-
		(Significant)	1	-	-
	Btu	(Best Fit)	3	-	-
		(Significant)	2	2	2
	Lbs S/MMBtu	(Best Fit)	-	1	1
		(Significant)	1	1	-
12	Sulfur	(Best Fit)	-	-	-
		(Significant)	-	-	-
	Btu	(Best Fit)	-	1	-
		(Significant)	1	1	1
	Lbs S/MMBtu	(Best Fit)	-	-	-
		(Significant)	-	-	-

Sheet 2 of 2

U.S.B.M. Producing District	Variable		Normal	Inverted Gamma	Log-normal
16	Sulfur	(Best Fit)	1	-	-
		(Significant)	-	-	-
	Btu	(Best Fit)	-	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	2	-	-
		(Significant)	2	2	1
17	Sulfur	(Best Fit)	1	-	1
		(Significant)	-	1	-
	Btu	(Best Fit)	-	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	1	1	-
		(Significant)	-	1	1
18	Sulfur	(Best Fit)	-	1	-
		(Significant)	-	-	-
	Btu	(Best Fit)	-	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	-	1	-
		(Significant)	-	-	-
19	Sulfur	(Best Fit)	1	4	-
		(Significant)	3	4	2
	Btu	(Best Fit)	3	1	-
		(Significant)	2	2	2
	Lbs S/MMBtu	(Best Fit)	3	2	2
		(Significant)	4	4	5
21	Sulfur	(Best Fit)	-	1	1
		(Significant)	-	-	-
	Btu	(Best Fit)	1	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	-	1	2
		(Significant)	-	1	1
22	Sulfur	(Best Fit)	-	-	1
		(Significant)	-	-	1
	Btu	(Best Fit)	-	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	-	-	1
		(Significant)	-	-	1
Total	Sulfur	(Best Fit)	15	17	8
		(Significant)	9	15	9
	Btu	(Best Fit)	17	9	2
		(Significant)	14	14	10
	Lbs S/MMBtu	(Best Fit)	14	15	14
		(Significant)	12	19	15

Based on the information in Table K-1, no firm conclusions can be made with respect to the best distribution for any of the three variables. The distributions exhibiting the best fit vary considerably from data set to data set. However, the normal distribution seems to be slightly better for the Btu values, while the sulfur content and lbs S/MMBtu appear to be best represented by the inverted gamma distribution.

1.2 Raw Coals, Top 1.5 Percent of the Distribution for Lbs S/MMBtu

Table K-2 presents the analysis of the best fit for lbs S/MMBtu for the top 1.5 percent of the distribution for raw coals. Part A of Table K-2 represents coals which were not sorted by lot-size, while Part B shows the same information for coals after the data were sorted by lot-size. In general, the coals in Part A exhibit a wide range of volumes (for example, 1,000 to 10,000 tons), while the coals in Part B are based on narrowly defined volume intervals (such as 1,000 to 2,000 tons). The choice for the best fit here is the lognormal, although it appears only slightly better than the inverted gamma. The normal distribution not only does not fit well, but also appears to be biased because it consistently underestimated the number of observations in the top 1.5 percent of the distribution. Thus, if one were interested in the top 1.5 percent, one should use either the inverted gamma or lognormal distribution.

1.3 Raw Coals, Top 15 Percent of the Distribution for Lbs S/MMBtu

Table K-3 shows the results of the analysis of raw coals for the top 15 percent of the distribution. Part A of the table is based on coals not sorted by lot-size, while

TABLE K-2

BEST FIT ANALYSIS FOR LBS S/MMBTU

(Raw Coals, Top 1.5 Percent of Frequency Distribution)

	Number of Data Sets Best Fit By:		
U.S.B.M. Producing District (1)	Normal (2)	Inverted Gamma (3)	Log-normal (4)
A. Not Sorted by Lot-Size			
1	-	2	4
4	1	6	5
8	-	1	2
9	1	1	2
10	-	2	-
11	-	1	-
17	-	1	-
18	-	-	1
19	1	1	3
21	-	1	1
22	1	-	1
Total	4	16	19
B. Sorted by Lot-Size			
1	2	3	2
4	-	1	4
8	-	2	1
9	1	2	1
19	1	-	1
21	1	-	-
22	1	-	-
Total	6	8	9

Source: Foster Associates, Inc.

TABLE K-3

BEST FIT ANALYSIS FOR LBS S/MMBTU

(Raw Coals, Top 15 Percent of Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Mines (2)	Average Relative Error (%) Normal (3)	Inverted Gamma (4)	Log-normal (5)	Best Fit Normal (6)	Inverted Gamma (7)	Log-normal (8)
A. Not Sorted by Lot-Size							
1	4	10.5	12.9	241.4	3	1	-
3	1	17.8	29.8	30.8	1	-	-
4	16	22.7	31.3	46.4	9.5	3	3.5
8	3	32.9	32.4	30.6	-	2	1
9	4	9.0	17.4	33.8	3	1	-
10	1	22.3	38.5	22.3	.5	-	.5
11	1	2.6	11.7	2.6	.5	-	.5
17	1	37.1	45.0	102.2	1	-	-
18	1	15.6	1.9	25.0	-	1	-
19	5	11.1	17.3	24.2	2.5	2	.5
21	2	17.1	29.7	1.5	.5	-	1.5
22	2	33.6	51.5	113.6	2	-	-
Total	41				23.5	10	7.5
B. Sorted by Lot-Size							
1	5	10.1	9.7	28.3	2	3	-
3	1	42.7	58.0	78.3	1	-	-
4	6	11.1	14.1	19.9	3	2	1
8	3	12.9	4.4	16.2	-	3	-
9	3	10.8	16.0	20.9	1.5	-	1.5
19	2	1.9	10.3	30.9	2	-	-
21	1	23.6	13.2	6.7	-	-	1
22	1				1	-	-
Total	22				10.5	8	3.5

Source: Foster Associates, Inc.

Part B shows the same analysis based on data sorted by lot-size.[1] Columns (3), (4), and (5) on Table K-3 set out the average relative errors for the relevant mines. Since the average relative errors could be biased by the presence of one particularly bad fit, the number of mines for which each particular distribution provided the best fit is also indicated in Columns (6), (7), and (8). The inverted gamma and the normal distributions provide the best fit for the top 15 percent. The normal distribution appears to be slightly better, but an insufficient number of data sets were analyzed to make this statement with any degree of confidence. Based on this analysis, the lognormal distribution clearly provides the worst fit of the three distributions for the top 15 percent of the frequency distribution.

1.4 Raw Coals, Top 1.5 Percent of the distribution for Heat Content

Set out in Table K-4 are the results of the best fit analysis for the heat content of raw coals in the top 1.5 percent of the distribution. Part A of Table K-4 is based on coals not sorted by lot-size, while Part B reflects data which were sorted by lot-size. In both instances there appears to be an indication that the normal distribution provides the best fit in the top 1.5 percent. However, because of the limited number of data sets, the evidence is not conclusive.

---

[1] It should be noted that when the data were sorted by lot-size (or interval analysis), the number of observations contained within a specific lot-size were in effect a subset of the data unsorted by lot-size. Frequently, the number of observations contained within the interval specified were insufficient for an examination of the right tail of the distribution.

TABLE K-4

BEST FIT ANALYSIS FOR HEAT CONTENT (BTU/LB)

(Raw Coals, Top 1.5 Percent of Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Data Sets Best Fit By: Normal (2)	Inverted Gamma (3)	Log-normal (4)
A. Not Sorted by Lot-Size			
1	1	2	1
3	1	-	-
4	5	2	2
9	2	1	1
10	-	1	-
11	2	-	-
19	1	3	-
21	-	1	-
22	1	-	1
Total	13	10	5
B. Sorted by Lot-Size			
1	3	1	-
3	-	1	-
4	2	2	2
8	1	-	1
9	-	-	1
19	2	-	-
21	-	1	-
22	-	1	-
Total	8	6	4

Source: Foster Associates, Inc.

1.5 Raw Coals, Top 15 Percent of the Distribution for Heat Content

Table K-5 shows the results of the analysis of the best fit for the heat content of raw coals for the top 15 percent of the distribution. Part A of Table K-5 is based on coals not sorted by lot-size, while Part B is based on data which were sorted by lot-size. From Table K-5 it can be seen that the normal distribution has a consistently lower average relative error than either the inverted gamma or lognormal distributions. Also, the analysis clearly shows that the normal distribution tends to fit the data better more often than any of the two alternative distributions.

1.6 Raw Coals, Top 1.5 and 15 Percent of the Distribution for Sulfur Content

Because sulfur content was not considered to be of as much interest as the lbs S/MMBtu variable, detailed analyses of the top 1.5 and 15 percent were not undertaken. However, literally hundreds of distributions were visually examined to determine which of the three distributions appeared to fit the top right tail the best. These visual examinations indicated that the percent sulfur can best be described by the inverted gamma distribution, which also provided the best fit for lbs S/MMBtu.

1.7 Summary of the Analyses for Raw Coals

In conclusion, the following recommendations are made with respect to the most appropriate choice of distributions for raw coals:

(1) For heat content, the normal distribution
(2) For lbs S/MMBtu, the inverted gamma distribution
(3) For sulfur content, the inverted gamma distribution

TABLE K-5

BEST FIT ANALYSIS FOR HEAT CONTENT (BTU/LB)
(Raw Coals, Top 15 Percent of Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Mines (2)	Average Relative Error (%) Normal (3)	Inverted Gamma (4)	Log-normal (5)	Best Fit Normal (6)	Inverted Gamma (7)	Log-normal (8)
A. Not Sorted by Lot-Size							
1	4	66.9	74.7	181.9	3	1	-
3	1	47.2	51.3	147.9	1	-	-
4	15	47.3	55.1	91.4	14.5	-	.5
8	3	46.8	49.8	153.9	2	1	-
9	4	25.2	28.7	42.9	3	-	1
10	1	32.5	40.0	32.5	.5	-	.5
11	1	103.6	110.9	148.9	1	-	-
17	1	264.0	280.0	506.7	1	-	-
18	1	575.0	610.0	-	1	-	-
19	5	107.3	110.8	846.0	3	2	-
21	2	31.5	39.6	123.1	2	-	-
22	2	8.4	9.5	306.1	1	1	-
Total	40				33	5	2
B. Sorted by Lot-Size							
1	5	41.1	45.5	183.6	4	1	-
3	1	33.8	36.9	52.9	1	-	-
4	6	42.5	51.1	73.7	6	-	-
8	3	14.1	25.8	21.6	1	2	-
9	3	24.2	27.3	67.7	2	1	-
19	2	14.3	16.9	820.6	2	-	-
21	1	6.7	3.9	5.0	-	1	-
22	1	17.5	15.0	2.9	-	-	1
Total	22				16	5	1

Source: Foster Associates, Inc.

2.0 Washed Coals

2.1 Washed Coals, Overall Goodness of Fit

Table K-6 presents the results of the chi-square analysis for washed or partially washed coals. The results are somewhat ambiguous but the best distribution for lbs S/MMBtu appears to be the inverted gamma or lognormal distribution. Heat contents seems to be best fitted by a normal distribution, and sulfur contents by either an inverted gamma or lognormal distribution.

2.2 Washed Coals, Top 1.5 Percent of the Distribution for Lbs S/MMBtu

Set out in Table K-7 are the results of the analysis of the distributions of lbs S/MMBtu for washed coals. Table K-7 indicates that for the top 1.5 percent of the distribution the best fit is provided by the inverted gamma, which appears superior to the lognormal.

2.3 Washed Coals, Top 15 Percent of the Distribution for Lbs S/MMBtu

Table K-8 presents the analysis of the top 15 percent for lbs S/MMBtu. The lognormal seems to be clearly less preferable than the inverted gamma or the normal distribution. With respect to these last distributions, there appears to be no significant differences in the case of the coals sorted by lot-size but in the coals not sorted by lot-size, the normal appears superior to the inverted gamma.

TABLE K-6

CHI-SQUARE ANALYSIS
WASHED OR PARTIALLY-WASHED COALS

U.S.B.M. Producing District	Variable		Normal	Inverted Gamma	Log-normal
1	Sulfur	(Best Fit)	1	-	-
		(Significant)	1	1	1
	Btu	(Best Fit)	1	-	-
		(Significant)	-	1	1
	Lbs S/MMBtu	(Best Fit)	-	1	-
		(Significant)	1	1	1
4	Sulfur	(Best Fit)	-	-	1
		(Significant	-	-	1
	Btu	(Best Fit)	-	1	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	-	1	-
		(Significant)	-	1	-
9	Sulfur	(Best Fit)	-	-	1
		(Significant)	-	1	1
	Btu	(Best Fit)	1	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	-	-	1
		(Significant)	-	-	1
10	Sulfur	(Best Fit)	-	3	2
		(Significant)	2	3	2
	Btu	(Best Fit)	2	2	4
		(Significant)	7	4	6
	Lbs S/MMBtu	(Best Fit)	1	3	2
		(Significant)	4	4	5
11	Sulfur	(Best Fit)	-	-	1
		(Significant)	-	1	1
	Btu	(Best Fit)	1	-	-
		(Significant)	-	-	-
	Lbs S/MMBtu	(Best Fit)	-	-	1
		(Significant)	-	-	1
15	Sulfur	(Best Fit)	-	-	-
		(Significant)	-	-	-
	Btu	(Best Fit)	-	1	-
		(Significant)	1	1	1
	Lbs S/MMBtu	(Best Fit)	-	-	-
		(Significant)	-	-	-
Total	Sulfur	(Best Fit)	1	3	5
		(Significant)	3	6	6
	Btu	(Best Fit)	5	4	4
		(Significant)	8	6	8
	Lbs S/MMBtu	(Best Fit)	1	5	4
		(Significant)	5	6	8

Source: Foster Associates, Inc.

TABLE K-7

BEST FIT ANALYSIS FOR LBS S/MMBTU

(Washed Coals, Top 1.5 Percent of the Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Mines (2)	Best Fit Normal (3)	Best Fit Inverted Gamma (4)	Best Fit Log-normal (5)
A. Not Sorted by Lot-Size				
1	2	1	1	-
4	1	-	1	-
9	1	-	1	-
10	9	1	4	4
11	2	1	-	1
Total	15	3	7	5
B. Sorted by Lot-Size				
1	2	-	1	1
4	2	-	2	-
7	1	-	1	-
8	3	-	1	2
9	3	-	2	1
10	2	-	2	-
Total	13	0	9	4

Source: Foster Associates, Inc.

TABLE K-8

BEST FIT ANALYSIS FOR LBS S/MMBtu

(Washed Coals, Top 15 Percent of the Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Mines (2)	Average Relative Error (%) Normal (3)	Average Relative Error (%) Inverted Gamma (4)	Average Relative Error (%) Log-normal (5)	Best Fit Normal (6)	Best Fit Inverted Gamma (7)	Best Fit Log-normal (8)
A. Not Sorted by Lot-Size							
1	2	9.4	12.1	30.8	1	1	-
4	1	23.9	14.8	23.9	-	1	-
9	2	14.3	17.0	10.7	.5	-	1.5
10	9	42.2	51.3	70.1	6	1	2
11	3	11.3	6.6	20.7	2	1	-
Total	17				9.5	4	3.5
B. Sorted by Lot-Size							
1	2	17.0	7.0	17.0	1	1	-
4	2	13.6	14.5	18.1	1	1	-
9	3	13.0	17.2	15.7	1.5	1	.5
10	2	78.9	75.5	133.3	1	1	-
7	1	1.4	12.8	18.3	1	-	-
8	3	18.7	10.8	16.4	-	2	1
Total	13				5.5	6	1.5

Source: Foster Associates, Inc.

2.4 Washed Coals, Top 1.5 Percent of the Distribution for Heat Content

The results of the analysis for the top 1.5 percent of the distribution for heat contents of washed coals are set out in Table K-9. As in the case of raw coals, the normal distribution appears to be most appropriate.

2.5 Washed Coals, Top 15 Percent of the Distribution for Heat Content

The results of the analysis of the top 15 percent of the distributions for heat contents of washed coals are summarized in Table K-10. As in the case of all previous analyses of heat content, the best fit appears to be provided by the normal distribution.

2.6 Summary of the Analysis of Frequency Distributions for Washed Coals

The analysis of the frequency distributions of washed coals produced results which were not significantly different from the raw coals. In previous sections of this report, it was found that compared to raw coals, the washed coals exhibited a lower mean lbs S/MMBtu as well as a lower RSD of lbs S/MMBtu. However, based on this analysis of frequency distributions, it appears that coal washing does not alter the shape of the statistical distribution.

In view of the results for washed coals, the choice of the type of distributions for raw coal characteristics is equally applicable to washed coals. Thus, it seems appropriate to make the following recommendations for both raw and washed coals:

TABLE K-9

BEST FIT ANALYSIS FOR HEAT CONTENT (BTU/LB)

(Washed Coals, Top 1.5 Percent of the Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Mines (2)	Best Fit Normal (3)	Best Fit Inverted Gamma (4)	Best Fit Log-normal (5)
A. Not Sorted by Lot-Size				
1	1	1	-	-
4	1	-	-	1
9	1	-	1	-
10	9	6	1	2
11	2	2	-	-
Total	14	9	2	3
B. Sorted by Lot-Size				
1	1	1	-	-
4	2	2	-	-
7	1	1	-	-
8	3	2	1	-
9	3	1	1	1
10	2	-	-	2
Total	12	7	2	3

Source: Foster Associates, Inc.

TABLE K-10

BEST FIT ANALYSIS FOR HEAT CONTENT (BTU/LB)

(Washed Coals, Top 15 Percent of the Frequency Distribution)

U.S.B.M. Producing District (1)	Number of Mines (2)	Average Relative Error (%) Normal (3)	Average Relative Error (%) Inverted Gamma (4)	Average Relative Error (%) Log-normal (5)	Best Fit Normal (6)	Best Fit Inverted Gamma (7)	Best Fit Log-normal (8)
A. Not Sorted by Lot-Size							
1	2	33.9	38.4	116.0	2	-	-
4	1	9.4	11.9	75.0	1	-	-
9	2	6.1	6.1	119.4	1	1	-
10	9	54.1	58.4	80.2	6	2	1
11	3	34.1	36.1	83.2	2	1	-
Total	17				12	4	1
B. Sorted by Lot-Size							
1	1	27.1	30.0	84.4	1	-	-
4	1	13.4	17.1	41.3	1	-	-
9	3	8.8	8.1	94.7	1	2	-
10	2	8.45	8.9	44.7	1	-	1
7	1	108.8	117.1	787.5	1	-	-
8	3	106.4	110.6	229.1	2	-	1
Total	11				7	2	2

Source: Foster Associates, Inc.

Coal Characteristic	Best Fit Distribution
Lbs S/MMBtu	Inverted Gamma
Heat Content (Btu/Lb)	Normal
Sulfur Content	Inverted Gamma